5. Mathematics up to the Secondary Level in India

Hridaykant Dewan[*], Namrita Batra[**] & Inder Singh Chabra[**]
[*]*Vidya Bhawan Society, Udaipur*
[**]*Vidya Bhawan Education Resource Center, Udaipur*
hardy.dewan@gmail.com, namrita.batra@gmail.com, inder@vidyabhawan.org

Background

India has placed great emphasis on educating all its children, since independence. Seeking a more just and equitable society, the Constitution of India is committed to providing to all children, opportunities for developing their capabilities and maximizing their learning in their areas of interest. Providing mathematics education is an integral part of India's commitment to universalization of education. Mathematics is a part of our general education and all children have to study mathematics till class 10.

Universalization of education was not an easy task for India at the time of independence. Large regions in the country did not have schools, schools that existed lacked infrastructure and the commonly held perception was that school education is not useful for all. Since then various initiatives of the government have led to a remarkable improvement in access to schooling and various studies show that demand for good schooling is not restricted to only certain groups of people today (PROBE Team, 1999). Significantly, the 86th constitutional amendment declared education a fundamental right of every Indian child in 2002, and the Right to Free and Compulsory Education Act (RtE) in 2009 gave further teeth to the idea of every child being educated up to the age of 14 (i.e. elementary school level) by making it justiciable. Today, primary schools exist within a kilometer of every child and elementary schools, every three kilometers. Access to secondary schools however, may require children to travel up to ten kilometers. While considerable progress has been made in providing schooling facilities to all children, children's learning remains a tenuous area. Various studies undertaken by government and private agencies in primary and elementary classes are evidence of very poor learning levels among children in both Language and Mathematics (Education Initiatives, 2010; Pratham, 2005-2010; NCERT,

2008). Children have difficulty in 'reading texts with understanding' and 'expressing their thoughts in writing'. Understanding of mathematics in primary classes is largely limited to 'procedural or rote-based learning' and in fact falling averages as we move from the primary to the elementary classes indicate an increase in the level of incomprehension for children (Education Initiatives, 2010).

In this paper, we will present how mathematics education, up to the secondary level, is conceptualized by our policy and curricular documents, textbooks and within the classroom. We will focus on the developments after 2005, but will spend some time discussing the journey. We will end with the challenges that exist for mathematics education on the road ahead.

Organization of secondary education in India

Education is a part of the federal framework of governance in India and so both the centre and the state governments enjoy authority in this area. The National Council for Education, Research and Training (NCERT) is the apex body for advising the central and state governments on school education. NCERT along with its state level counterparts– the State Councils of Education, Research and Training(SCERTs) are involved in various tasks like educational research, curriculum renewal, textbook creation, creation of supplementary material for children and teachers, pre- and in-service training and publications for teachers and children.

The country also has two national level boards of secondary education- the Central Board of Secondary Education (CBSE) and National Institute of Open Schooling (NIOS), the former being popular. All states of the country also have their own official boards of secondary education. Apart from these, one private board of secondary education also exists–the Council for the Indian School Certificate Examination (CISCE). More recently some international boards of secondary education are also coming into India. The secondary boards in India are in many cases responsible for the development of curricular expectations, syllabus and teaching-learning materials at the secondary level as well as reform in examination and evaluation practices. In a few instances they are also responsible for in-service teacher training.

In recent years, there has been an increase in the role of the NCERT and the SCERTs in processes of curriculum renewal and textbook development and boards are focusing on improving assessment processes and mechanisms.

It must be kept in mind that while there are official agencies like NCERT, SCERT and boards of secondary education that produce textbooks of mathematics, there are no restrictions on private publishers bringing out materials for these classes. There are many national and international publishing houses that bring out books; however, the

expectation is that these be in line with the national and state curricular documents. Private schools largely form the market for these books.

The vision for mathematics education

The vision with which mathematics has been placed in the school curriculum has evolved over the years. In the 1950s and the 1960s, India developed its mathematics education as a step towards industrialization and scientific research. The Kothari Commission was set up for thinking comprehensively about education in India during this period and published its report in 1966. The report underlined the need for mathematics and science in school as well as in higher education; it emphasized the importance of children learning mathematics for the development of science and technology and for industrial growth. To quote from the report, "One of the outstanding characteristics of scientific culture is quantification. Mathematics, therefore, assumes a prominent position in modern education. Apart from its role in the physical sciences it is now playing an increasingly important part in the development of the biological sciences" (Government of India–Ministry of Education, 1966, p.181). The 1968 and 1986 National Policies of Education spoke in the same tone as the Kothari Commission report and the 1986 policy states that "mathematics should be visualized as the vehicle to train a child to think, reason, analyze and articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning" (Government of India–Ministry of Human Resource Development, 1998, p.29).

The system of Nai Talim (New Education) that had emerged in the 1930s and 1940s from the thinking of various people like Dr. Zakir Hussain and Gandhi working towards building responsible, capable and educated Indians also realized the importance of mathematics. However, it viewed mathematics in terms of its use for the day-to-day requirements of people. The emphasis was on ensuring that calculations necessary for the survival of the child in the circumstances in which she was growing were learnt. The Zakir Hussain committee stated: "Knowledge of mathematics is an essential part of the curriculum. Every child is expected to work out the ordinary calculations required in the course of his craft work or his personal and community concerns and activities." In this sense, the Kothari Commission widened this very concrete, tangible and narrow purpose of teaching mathematics.

Respecting the distribution of the areas of jurisdiction between the centre and state governments on matters of education, the National Policy of Education, 1986 clearly states that "the national system of education will be based on a national curricular framework which contains a common core along with the other components that are flexible". The national curriculum framework brought out by the NCERT in 2000 gave some idea of the content in the syllabus and the kind of teaching process to be followed.

It felt that the teaching-learning process must heed the context of the child and their 'zone of proximal development' and learners should be able to relate the mathematics in their textbooks to their life experiences. This led to the idea of the mathematics lab and use of more and more concrete illustrations and activities in classrooms of mathematics. Under central government supported schemes, teachers and teacher educators made a lot of effort to develop activities and games that would somehow be linked to the teaching of mathematics.

The period of the 1990s and early 2000s was also the period when Minimum Learning Levels (MLL) formed the basis for the curriculum and textbooks and NCF 2000 also asked for their proper implementation. The idea of MLL arose from the need to provide equitable education to all children across India. It itemized learning of language, mathematics and environmental studies in the primary classes into small chunks/ competencies that all children were expected to achieve. Assessment and evaluation was also based on these small chunks. To be measurable these competencies had to be in the form of observable behaviour demonstrated by the child when she received the requisite inputs. This formulation of MLL also paid no heed to the time and space that children needed for concept building. There was a great deal of opposition to this and various alternative formulations were built. These included work by some organizations outside the government framework, some of them being partnerships with public institutions, like Eklavya in Madhya Pradesh, Homi Bhabha Centre for Science Education in Maharashtra, Vidya Bhawan Society in Rajasthan, Suvidya in Karnataka, School Mathematics Program of the Centre for Science Education and Communication of Delhi University, etc. These organizations had worked directly with various government schools and developed their own curriculum, syllabus and textbooks in this process. The experiences and ideas of these organizations have helped in giving shape to the National Curriculum Framework 2005. In fact, the upper primary textbooks produced by the Delhi state in 2000 were also a partnership between SCERT, Delhi and Vidya Bhawan Society, Rajasthan.

In the exercise undertaken by Delhi SCERT, many conceptual areas were re-organized and books made less loaded, complicated calculations eschewed and many areas elaborated. Topics such as surds, complicated proofs, stocks and shares, dividend calculations, income and sales tax were not included. The textbooks also attempted to use language and pictures as devices to communicate mathematics and were based on the argument that a book for the student should be at the level of her comprehension. Another important change initiated was the creation of a complete mathematics book instead of a textbook divided into sections. This subsequently led to spiraling and developing inter-relationships between various mathematics concepts. There was, however, no consensus on removing relatively tedious algebraic expressions, fractional number calculations, theorems and definitions in geometry, etc. There was a fear that the state syllabus would lag behind that of other states across the country. It was difficult for many to accept

that it was pointless to load the program with tricks and algorithms to solve particular problems or for the child to do tedious algorithmic manipulations with numbers, algebraic quantities or geometric figures.

All this was part of the wisdom that fed into the emergence of the next National Curriculum Framework in 2005.

Mathematics education and the National Curriculum Framework (NCF) 2005

The National Curriculum Framework 2005 along with its Position Paper on Teaching of Mathematics published in the subsequent year provides direction to school and teacher education all across India, currently.

The main goal of mathematics education emphasized in NCF 2005 is 'mathematization' of the child's thought and processes. In doing so the document visibly enlarges the vision of school mathematics taking it beyond areas of obvious utility in daily life to enriching a student's scope of thought and visualization and in turn her ability to relate to the world and to mathematics. It talks about teaching mathematics not just for utilitarian purposes and recognizes mathematics as an important part of the development of the human mind, as an addition to the human ability to absorb, visualize, logically understand, build arguments, prove statements and in a sense interact with and deal with the world. To quote the document, "Developing children's abilities for mathematization is the main goal of mathematics education. The narrow aim of school mathematics is to develop 'useful' capabilities, particularly those relating to numeracy – numbers, number operations, measurement, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems." (NCERT, 2005, p. 42)

The curriculum at the primary stage emphasizes concrete experiences (objects and day-to-day life experiences) in the progression towards mathematical abstraction. It gives due place to non-number areas of mathematics like space, visual patterns and data handling. While discussing number areas it recommends development of number sense including number patterns and de-emphasizes the algorithm, encouraging children to develop their own methods of solving problems. It curbs the need to teach children bigger and bigger numbers to prevent "overloading the child's cognitive capacity which can be better used for mastering the logical skills at the earlier stages" (NCERT, 2006, p.15). It provides space to visualization, estimation and reasoning along with computation abilities. It also recommends building a stronger conceptual base for fractions and decimals and de-emphasizes operations with fractions at the primary level.

At the upper primary stage, concepts that children have learnt are re-visited in more abstract forms, are consolidated and are elaborated into denser ones. Arithmetic is extended to algebra and children are expected to express the patterns they are seeing through generalizations. A study of space is undertaken through a Euclidean study of triangles and quadrilaterals as well as through solid geometry. Here the emphasis is on mathematical thinking and visualization. Data handling is also an essential area at this stage.

The broad description of the purpose of mathematics for secondary classes includes consolidating and elaborating the conceptual edifice of the elementary classes and significantly building upon the ability to perceive rules and generalizations, formulate ideas with precision, the ability to make logical arguments and the ability to prove statements. The secondary school books build on the pattern recognition and generalizations in the elementary books and go on to problems that require proofs to be found. These are simple to prove and can be done using many strategies. The logical formulation and the arguments included in each step along with precision of presentation are of value to engage with the world more effectively.

In the context of universalization of education the position paper on mathematics importantly talks about the development of a mathematics program that would ensure that everybody learns mathematics and does not fear it. The document identifies children's inability to deal with the hierarchical nature of mathematics as one of the main reasons for their giving up on it and thus emphasizes that the progress of the syllabus should be such that children have sufficient time to develop the fundamental concepts and thereby do not feel afraid of moving ahead. It also strengthens the need for the mathematics program to be so designed that it takes into account the requirement of revisiting concepts. The concepts in the program are sought to be developed spirally, with each concept introduced and dealt with on many occasions to give repeated opportunities to the learner to absorb them. Another reason that the position paper points to while discussing the fear of mathematics is the manner in which that the "language of mathematics learnt in school is far removed from their everyday speech, and easily forbidding" (NCERT, 2006, p. 5). The curriculum therefore also expects that the language used in textbooks would be like that spoken by children in their daily life.

Understanding the importance of the relationship between language and learning the curriculum framework emphasizes that the mathematics classroom should be alive and interactive in which children should articulate their own understanding of concepts, evolve models and develop definitions. Following the recommendations of the position paper on mathematics developed as a part of NCF 2005, the subsequent books for the elementary and secondary classes provide various opportunities to the learners to formulate principles and solutions in their own words. They argue that this helps develop

and consolidate conceptual frameworks. The program also emphasizes the role of dialogue among learners and argues for opportunities for children to discuss and make presentations as a group.

Another principle that the curricular document lays down is that learning mathematics is not about remembering solutions or methods but about feeling capable of and knowing how to solve new problems. It also realizes the importance of problem posing in mathematics. The twin tasks of solving and setting problems helps develop an understanding of the concepts and principles of mathematics.

Importantly, it also asks for a need to look at mathematics as a whole and not through water tight sections of arithmetic, algebra and geometry; in doing so it advocates making connections within these areas of mathematics. The understanding is also that better conceptual understanding of mathematics involves exploring the relationship that mathematics has with other subjects as well. The mathematics program therefore makes an effort to link itself to other areas and have problems that include concepts from them. The effort is to help learners get a wider and deeper sense of mathematics and make them confident of dealing with its basic ideas. In the mathematics program up to class X, relationships with the natural and physical sciences, economics, etc., have been explored.

NCF 2005 thus advocated a major re-look at the syllabus, textbooks, nature of assessment and more importantly the way mathematics was taught in classrooms.

The syllabus after NCF 2005

Here we will discuss the prominent changes in the NCERT syllabus for the elementary and the secondary classes. The syllabus has undergone some substantial changes in terms of areas as well as the nature of treatment.

The presentation of algebra, geometry and data handling have changed considerably following the NCF 2005. Algebra has changed from learning to handle complicated algebraic expressions, using algorithms and learning to match them to specific problems involving tedious calculations in de-contextualized settings. It has become instead an attempt to understand the idea of a variable, functional relationships and the use of letter numbers in different ways. The attempt is to now understand algebra as a generalization of many of the ideas that are intuitively learnt or seen as patterns and recognizing that all the ideas that are intuitive or obtained from patterns cannot be generalized. It is inextricably linked to learning the use of mathematical language, of symbols and brevity.

Geometry has changed from remembering theorems and proofs to understanding space and spatial relations. Theorems and proofs are a part of understanding shapes and their properties along with the ability to use mathematical logic. Geometry as a whole now relates to visualization, symmetry, representation of objects, projecting and mapping,

plotting functional relationships. Its relationship with algebra and arithmetic is therefore much clearer and elaborate. Some examples of what is included now are: solid geometry as an exercise in 3D visualization and conveying the idea that Euclidean geometry is not the only form of geometry, change in the handling of solid shapes through formulas of surface area and volume to understanding them through nets, developing an understanding of edges, surfaces, vertices, etc., and the ability to imagine objects from different positions and perspectives.

Data handling has emerged as an introduction to statistics. The earlier view was that data handling cannot be initiated before secondary classes. The extension of data handling to collection, organizing and presenting data through pictograms, tables, bar graphs and pie charts has made it possible to introduce it much earlier. Data handling is no longer about only calculating representative values like mean, median mode but about understanding when we need to use which representative value.

There have been significant shifts in the secondary classes as well. For example the logarithmic and trigonometric tables have not been included in NCERT books indicating thereby that there is no expectation from the child to do complicated calculations using these. The details of commercial mathematics have been reduced and emphasis changed to helping children understand the underlying concepts of ratio and proportion and linking different examples of their use under one conceptual thread. The nature of geometry has changed from a lot of theorems and knowing their proofs to development of an understanding of concepts using their experience and helping them understand the notion of a proof and how to construct it. The extent of work expected on circles has been considerably reduced in order to deepen and widen conceptual ideas on polygons. For equations, the importance and meaning of roots through graphical representation and factorisation is emphasized. There is an effort to help students form an idea of functional relationships.

The expectation is thus that mathematics emerges as a subject of exploration and creation rather than as an exercise of finding old answers to old and complicated problems.

Textbooks after NCF 2005

If we look at NCERT books before and after NCF 2005 we see a marked difference in their presentation and appearance.

The principles that these books have utilized are –

- Textbooks should link the mathematics that children do in their textbooks to the mathematics they see and experience all around. Concepts should be introduced through situations of life in which they are placed.

Whereas mean gives us the average of all observations of the data, the mode gives that observation which occurs most frequently in the data.

Let us consider the following examples:

(a) You have to decide upon the number of chapattis needed for 25 people called for a feast.

(b) A shopkeeper selling shirts has decided to replenish her stock.

(c) We need to find the height of the door needed in our house.

(d) When going on a picnic, if only one fruit can be bought for everyone, which is the fruit that we would get.

In which of these situations can we use the mode as a good estimate?

Consider the first statement. Suppose the number of chapattis needed by each person is 2, 3, 2, 3, 2, 1, 2, 3, 2, 2, 4, 2, 2, 3, 2, 4, 4, 2, 3, 2, 4, 2, 4, 3, 5

The mode of the data is 2 chapattis. If we use mode as the representative value for this data, then we need 50 chapattis only, 2 for each of the 25 persons. However the total number would clearly be inadequate. Would **mean** be an appropriate representative value?

For the third statement the height of the door is related to the height of the persons using that door. Suppose there are 5 children and 4 adults using the door and the height of each of 5 children is around 135 cm. The mode for the heights is 135 cm. Should we get a door that is 144 cm high? Would all the adults be able to go through that door? It is clear that mode is not the appropriate representative value for this data. Would **mean** be an appropriate representative value here?

Why not? Which representative value of height should be used to decide the doorheight?

Similarly analyse the rest of the statements and find the representative value useful for that issue.

Figure 1: Data Handling, Class-7 NCERT Textbook

- Concepts and ideas should also be arrived at by observing patterns, exploring them and providing children opportunities to define them in their own words. Definitions and excessive terminology cannot be the beginning of concept formation.
- Concept building should be followed up with examples and exercises. Exercises should give practice (in both concepts and process) in various contexts. Exces-

THINK, DISCUSS AND WRITE

1. Can two adjacent angles be supplementary?
2. Can two adjacent angles be complementary?
3. Can two obtuse angles be adjacent angles?
4. Can an acute angle be adjacent to an obtuse angle?

Figure 2: Lines and Angles Class-7 NCERT Textbook

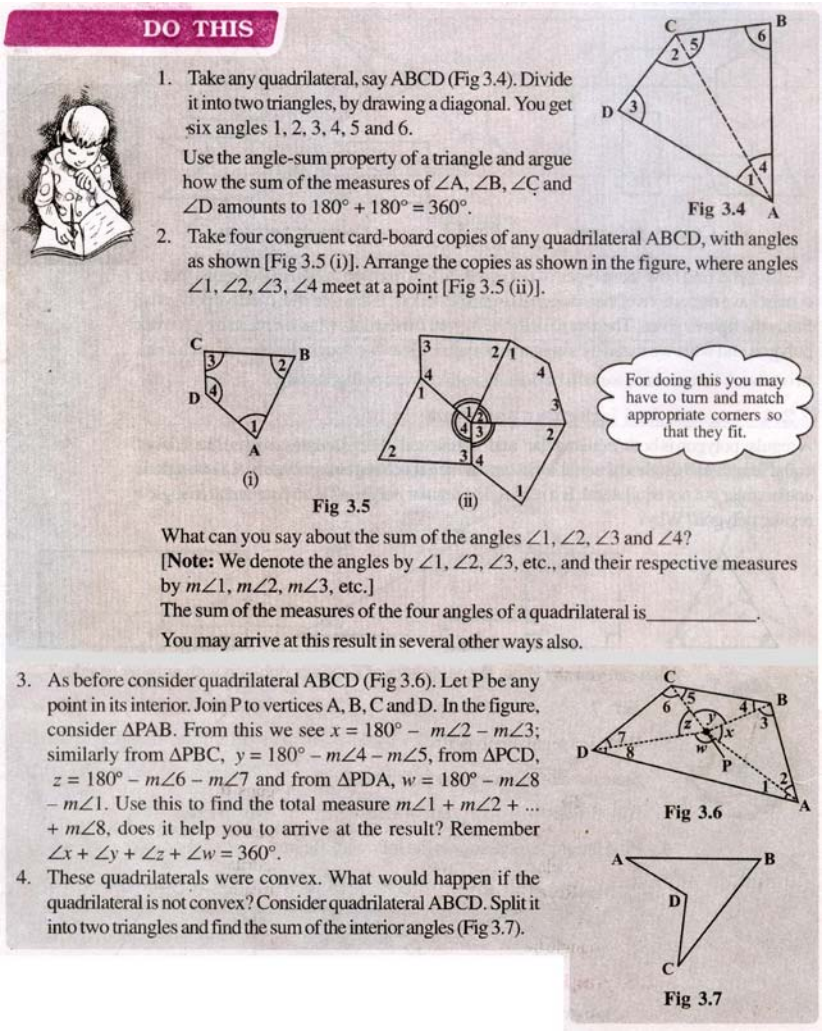
sive practice of algorithm should be avoided. These exercises should not only be placed at the end of a chapter but smaller ones should be present at different points when it is felt that some thought or practice is needed.

- Textbooks should be able to establish continuity with what children have previously learnt in the topic through a spiral arrangement.
- Wherever possible problems should be solved using more than one method. Children should also be encouraged to do the same and also come up with their own ways of solving problems.
- Problem posing is an important part of math and children should be encouraged create a variety of problems.
- Challenging questions should be provided at the end of each chapter.
- Textbooks should give space for collaborative learning and give space to children to work in groups and in pairs.
- Textbooks should use language which a child would normally speak and understand. As far as possible they should act as self-learning material for the student.
- Pictures should be used thoughtfully. They could be used to help the child in concept building and should also be used as background fillers to convey the idea that mathematics can be fun, can be done collectively and the math classrooms can be organized in many creative ways. The fillers also show that the book is for the child and she needs to think, solve problems and figure out ways.

Figure 3: NCERT class 7 and 8 textbooks

NCERT Class 7 and 8 textbooks

- Children are generally introduced to proofs in geometry. But it is important for them to understand that a similar process is followed for numbers and thus should be introduced to proofs in number theory too.
- All proofs need to be given in a non-didactic manner, allowing the student to see the flow of reason. Wherever possible more than one proof is to be given.



DO THIS

1. Take any quadrilateral, say ABCD (Fig 3.4). Divide it into two triangles, by drawing a diagonal. You get six angles 1, 2, 3, 4, 5 and 6.

   Use the angle-sum property of a triangle and argue how the sum of the measures of ∠A, ∠B, ∠C and ∠D amounts to 180° + 180° = 360°.

Fig 3.4

2. Take four congruent card-board copies of any quadrilateral ABCD, with angles as shown [Fig 3.5 (i)]. Arrange the copies as shown in the figure, where angles ∠1, ∠2, ∠3, ∠4 meet at a point [Fig 3.5 (ii)].

Fig 3.5

For doing this you may have to turn and match appropriate corners so that they fit.

What can you say about the sum of the angles ∠1, ∠2, ∠3 and ∠4?

[**Note:** We denote the angles by ∠1, ∠2, ∠3, etc., and their respective measures by $m\angle 1$, $m\angle 2$, $m\angle 3$, etc.]

The sum of the measures of the four angles of a quadrilateral is_____.

You may arrive at this result in several other ways also.

3. As before consider quadrilateral ABCD (Fig 3.6). Let P be any point in its interior. Join P to vertices A, B, C and D. In the figure, consider ΔPAB. From this we see $x = 180° - m\angle 2 - m\angle 3$; similarly from ΔPBC, $y = 180° - m\angle 4 - m\angle 5$, from ΔPCD, $z = 180° - m\angle 6 - m\angle 7$ and from ΔPDA, $w = 180° - m\angle 8 - m\angle 1$. Use this to find the total measure $m\angle 1 + m\angle 2 + ... + m\angle 8$, does it help you to arrive at the result? Remember $\angle x + \angle y + \angle z + \angle w = 360°$.
4. These quadrilaterals were convex. What would happen if the quadrilateral is not convex? Consider quadrilateral ABCD. Split it into two triangles and find the sum of the interior angles (Fig 3.7).

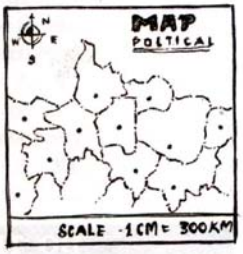
Fig 3.6

Fig 3.7

Figure 4: Understanding Quadrilaterals, Class-8 NCERT Textbook

Mathematical concepts should be used in tandem with concepts of other subjects to build a deeper understanding of mathematics.

Example 5: The scale of a map is given as 1:30000000. Two cities are 4 cm apart on the map. Find the actual distance between them.



Solution: Let the map distance be x cm and actual distance be y cm, then

$$1:30000000 = x : y$$

or $$\frac{1}{3\times10^7} = \frac{x}{y}$$

Since $x = 4$ so, $$\frac{1}{3\times10^7} = \frac{4}{y}$$

or $$y = 4 \times 3 \times 10^7 = 12 \times 10^7 \text{ cm} = 1200 \text{ km}.$$

Thus, two cities, which are 4 cm apart on the map, are actually 1200 km away from each other.

5. A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the *actual* length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?

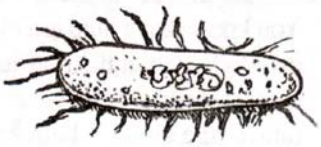


Figure 5: Direct And Inverse Proportions, Class-8 NCERT Textbook

- The texts and visuals should be sensitive to concerns of gender and equality.
- Anecdotes about the history of mathematics and achievements of mathematicians should be added make the subject interesting for children.
- The textbooks should also have some space to talk to the teachers about the design of the syllabus, the structure and presentation of the textbook including the exercises and on how to engage students.

As a part of the federal framework, states have jurisdiction to develop their own curriculum and textbooks keeping in mind the core areas laid out by the national curriculum. Thus, the impact of NCF 2005 is visible in textbook writing of a number of Indian states post 2005. Some experiences from textbook writing processes are mentioned here.

The state of Kerala has taken this spirit much farther, by taking up an extensive participatory process for curriculum formulation as well, in which literally thousands of mathematics teachers took part over a year. The new development of making textbooks available in electronic form, online or offline, has led to interesting possibilities unexplored hitherto. In the state of Kerala, software tools like Dr Geo and Geogebra have been integrated into these textbooks, so that students and teachers can use textbook illustrations interactively and dynamically, changing them as they grapple with meaning. Kerala's IT@School

project has also offered linkages by showing the use of these mathematical tools in Science textbooks, thus offering children an opportunity to connect mathematics with physics or chemistry textbooks.

Andhra Pradesh has developed a curriculum framework and series of position papers including the position paper on teaching of mathematics. This paper was based on the NCERT Position Paper on Mathematics and also included some new ideas on the specific concerns of the state. A similar exercise was undertaken by the Bihar state when it wrote its curriculum framework, including ideas on mathematics education. While Bihar has developed its mathematics textbooks up to Class 8, Andhra Pradesh is in the process of doing so. In both the states the process of curriculum and syllabus writing and the development of textbooks has been a joint effort of SCERT functionaries, teacher educators, and mathematics teachers of the state and a non-governmental organization, which has been involved in the development of National Curriculum Framework 2005 and NCERT textbooks.

This process has been a challenging one and has been a learning experience for all those involved in it. It has been challenging for various reasons.

Probably the most important one is the understanding of government functionaries, be they teachers, teacher educators or administrators of education, about 'why mathematics needs to be taught which in turn determines their choice of what should be taught'. Exposed to years of working with an over burdened syllabus of mathematics aspiring to teach children bigger numbers and a taller mathematics, with an emphasis on computation, algorithms and the 'correct method' and most likely a similar experience in their own education, many functionaries find it very difficult to unburden the syllabus and emphasize the process of mathematical thinking. Even though the NCF is very clear on this issue, state functionaries continue to feel that reducing topics leads to loss of mathematical knowledge and children of their state are being deprived in this process. They also feel that such reductions will make their children unfit for various competitive examinations that they will take at the end of schooling.

Another very real challenge for this group is that of 'actually writing' keeping in mind that we have to help the learner engage with concepts, associate it to their life and at the same time develop a capacity to handle abstraction. While writing the group always finds it easier to give information to children, lay out definitions, solve example questions and then give long exercises. The decision of when to move away from concrete objects as aids in understanding towards more abstract conceptualizations is also one that textbook writers have to engage with. Spiraling through concepts also does not come easily as textbook writers often feel that once they have dealt with a concept in a particular class, they need to basically test children for understanding in the next one and no concept revisiting is required. Even while voicing the progressive principles laid out in the

curriculum, textbook writers time and again fall back into their old writing styles and privilege mathematical knowledge over mathematical thinking. Giving space to children to work in pairs and in groups through tasks which have potential for collaboration is also an inculcated habit.

Another challenge is of building a healthy atmosphere of listening to feedback that other members of the group give about one's writing. The group of people who sit together also come with different experiences with different amounts of classroom experiences. Some are a part of the government structure and some are voices from outside. Writers also hold what they have written very close to their heart and find it difficult to take critical feedback and view their writing in the light of the set of logical principles that they have themselves laid down in the light of the curriculum. All these also present challenges but at the same time lead to richer and deeper discussions.

The task of the non-governmental partners in this endeavor has been to help the group retain focus on the principles of the NCF and help them write in a manner that children feel confident in approaching and continuing with mathematics. Importantly, it is also to build a healthy workspace where people are listening to each other's ideas and suggestions and owning the whole book and not only some chapters that they are involved in. This is a capacity building process for all involved. People learn a little bit more about mathematics and about mathematics teaching.

Clearly the goals of NCF 2005 are where we want to go. We are yet far from it and have earnestly started on the issues raised in the document. These are however very nascent attempts and the road ahead is long.

Challenges on the road ahead

The vision of mathematics education in NCF 2005 demands changes from the system and schools. It demands a change in the syllabi and textbooks and a change in classroom teaching and assessment. As we have discussed earlier, processes for the former have been initiated and stand at different levels of maturity in different states. However, the latter remains a formidable challenge. An appreciation of what NCF 2005 is saying requires extending the horizons of schools and linking them to the outside world and a different relationship between teachers and children including providing children with opportunities to explore, extend their mind and argue their stance. All these are very hard to achieve. There is little appreciation or acceptance of these principles in society, and among teachers and teacher educators, who are themselves struggling with their limitations in mathematical ability. Also there is little conviction that equitable learning is possible. The belief systems and prejudices about gender, caste, economic status and even cultural practices make mathematics teachers build classrooms differently from

those expected in the NCF.

The biggest challenge for us is to change this attitude of teachers, parents and others to mathematics and why and how it should be taught. For most people "why mathematics education" still revolves around mathematics for calculations. Generally, the teacher believes that mathematics is about knowing solutions to problems and not about being able to understand what the concept means and about being able to think of ways of solving problems. The emphasis is on the 'correct answer' rather than on thinking of a variety of ways to approach the solution. Teaching, therefore, gets restricted to sharing solutions with students from either the textbooks or guide books, which offer short cuts and memory devices to children and are used widely especially in the higher classes. Teachers teach in a manner that is entirely de-linked from the experiences of children and participation by children is minimal. There is often even confusion between 'demonstration through concrete examples' and 'the proof of statements'. For the students, the classrooms largely consist remembering the definitions of mathematical ideas, axioms, postulates and solutions to problems or theorems and their proofs. Mathematics classroom, therefore, tends to become uninteresting for students. For most teachers, making mathematics interesting and vibrant is not possible because they themselves are often afraid of mathematics and consider it a subject for the privileged few who are capable and intelligent. 'Activity based mathematics teaching' and 'child centered' teaching are the buzz words, open to multiple interpretations and often get restricted to use of concrete materials for a few concepts in primary classes. Mathematics classrooms, in spite of NCF and the recent textbooks of NCERT remain didactic and assessments test calculations, algorithms, definitions and answers to 'difficult questions'.

Teachers who teach mathematics at the elementary and the secondary level are supposed to be graduates or post graduates in mathematics with a degree/diploma for teaching. In many cases, however, teachers with such qualifications are not available to teach mathematics. Mathematics is taught by teachers who are not very confident of their mathematics. Even in cases where mathematics graduates or post graduates teach the subject their conceptual understanding may be inadequate. Besides, their understanding of the nature of mathematics and attitude to it and its learning are very different from what is underlined in the NCF 2005. The lack of ability of teachers in mathematics is probably the result of their preparation at the school and the college level. It is also because of the inadequate time for pre-service training and the way classroom teaching for pre-service teacher education takes place. Given the large number of teachers in schools and the lack of avenues of for continuing their learning most teachers also do not remain in touch with what they have learnt. There is a strong need for such processes to be initiated that would enable teachers to become more confident and to continue to engage with them. There are, however, insufficiently many institutions and individuals capable of creating and implementing a process that would enable teachers to learn more mathematics and be

more confident of their ability. In the Indian context, the lack of this institutional capacity to help teachers learn more mathematical concepts and more about mathematics is the biggest challenge. In India's effort towards universalization of mathematics education, these remain the most critical barriers. They affect the confidence and learning of children much more than the syllabus, textbooks, assessment and everything else put together.

A number of studies and experiences show that many barriers to schooling still exist. These include barriers for the girl child who is not allowed to go to the school after she has reached a certain age, generally the age of puberty. Many schools do not have boundary walls (52%) and separate toilets for girls (41%), and this takes schooling a step further away (NUEPA, 2009). The situation for the secondary classes is worse as the schools are farther from their homes and concerns about the security of girls, forces them to give up schooling. Another factor preventing girls from coming to school is the absence of women teachers in the higher classes. Access is not the only problem for girls and the general societal belief (also shared by teachers) is that the study of abstract ideas does not benefit girls and also that a girl's life priorities do not require her to take on anything as hard as mathematics and science. Frequently heard statements could be that "X is just like a boy, she is so good in mathematics". This attitude adds to the belief already implanted in them that they cannot learn mathematics.

There are also very strong prejudices about poor children and children from deprived social backgrounds. Some time ago almost all children in school were from the so called upper castes. The situation has changed today but a majority of mathematics teachers are still from the higher castes. Their belief is that the poor and lower caste children are not meant to learn mathematics and any sign of their disability is proof of their belief. It may not be hard to appreciate that such attitudes would also be present in children. Children from privileged backgrounds start with this advantage and that initial advantage is further strengthened by the belief of the system that only children from certain backgrounds can do abstract learning. This belief is in contrast to the commitment that India is bound to educate all its children and wants to teach mathematics to all children.

The NCF entails an expectation of a classroom that is interactive and inclusive and a teacher development program that not only builds the capability of the teachers for all this but also motivates them for this through mechanisms of sharing and scaffolding. At present various mechanisms for building the capabilities and interests of teachers are being evolved and include restructuring of pre-service courses of teacher education, strengthening of in-service training as well as strengthening of decentralized (cluster and block level) structures, seeking linkages between colleges of higher education and departments of education and teacher training colleges, etc. Attempts are also being made to reach ideas to the teachers through the use of ICT.

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