6. The Senior Secondary Mathematics Curriculum

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Introduction

The National Curriculum Framework (NCF) 2005 in its position paper on the 'Teaching of Mathematics' (National Council for Educational Research and Training [NCERT], 2005) describes the higher secondary stage as the "launching pad from which the student is guided towards career choices." At this stage the student has to make a choice as to whether she will opt for the science, commerce or humanities stream. Clearly mathematics has an important role to play here in developing her skills so that she may pursue her chosen course. For curriculum makers the most difficult choice at this stage is between breadth and depth. Whether the curriculum should offer exposure to a variety of topics from various areas or limit the number of topics to develop competence in a few areas is an issue for debate. According to Thurston, "Instead, there should be more courses available……which exploit some of the breadth of mathematics, to permit starting near the ground level, without a lot of repetition of topics that students have already heard."

The NCF suggests that if breadth is chosen over depth, then the decision as to the extent to which the topics should be developed is a matter of serious consideration. The topics which have importance for mathematics as a discipline should be included and their treatment should be done at least to the extent that the student is able to see the relevance or utility of those topics in mathematics or in some other course of study.

Content and structure of the curriculum: Brief commentary

The higher secondary mathematics curriculum is dominated by Differential and Integral Calculus accounting for almost half of the content in class 12. Other topics include Matrices and Determinants, Vector Algebra, Three Dimensional Geometry, Linear Programming and Probability. These topics remain isolated and there are few instances where the linkages across these topics are highlighted. Also manipulative and computational aspects of these topics, rather than applications, dominate mathematics at this stage.

The syllabus of class 11 includes important topics like Sets, Relations and Functions, Logic, Sequences, Series, Linear Inequalities, Combinatorics, Trigonometric Functions, Complex Numbers, Straight lines, Conic Sections and Statistics. The striking thing about the class 11 syllabus, in contrast to that of class 12, is its large number and variety of topics. While many of these topics are rich in mathematical content their treatment is only done at a surface level. Also, since the Board Examinations at the end of class 12 tests only the topics of class 12, these topics remain neglected. NCF 2005 recommends that curriculum designers reconsider the distribution of content between classes 11 and 12.

The NCF 2005 position paper begins by stating that the primary goal of mathematics education is the "mathematisation of the child's thought processes" and the development of the "inner resources of the growing child." Mathematics empowers an individual to think logically, handle abstractions, generalize patterns and solve problems using a variety of methods. The document states that for children to acquire such integrated skills, a curriculum is needed that is "coherent and teaches important mathematics"; here, 'coherence' refers to the way the different strands of the curriculum reinforce one another and enable the student to apply concepts learnt in one strand to other strands, and to other school subjects such as science and social studies. Also, the mathematics taught in school should be 'important' in the sense that "teachers and students find it worth their time … addressing [the] problems, and mathematicians consider it an activity that is mathematically worthwhile." In this context, the document recommends that mathematics teaching at all levels be made more 'activity oriented' and student centred, so that students understand the basic structure of mathematics and learn how to think mathematically and how to relate mathematics to life experiences.

The NCF 2005 recommendations have been the driving force for revisiting and revamping the elementary school mathematics curriculum. But the recommendations have had little impact on the senior secondary curriculum. The textbooks as well as the content of the senior secondary curriculum have undergone very few changes over the years. Some topics have been removed while others have been added, but the approaches to the topics have remained the same. In the textbooks, chapters include an introductory note with some historical background, the basic concepts, theorems, results, examples and exercises. However, there are very few inputs in terms of applications. For example, in the chapter 'Application of Derivatives' the topics covered are almost the same from 1989 to the present: Motion in a Straight Line, Motion under Gravity, Rate of Change, Increasing and Decreasing Functions, Maxima and Minima, Rolle's Theorem, Mean Value Theorem, Tangent and Normal, and Differentials and Approximations. Here are two typical problems from the textbook:

> A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.
>
> A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without a lid, by cutting off a square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is a maximum?

The emphasis is largely on developing manipulative skills to solve problems, and there is relatively little on visualizing concepts and exploring applications. Also, many questions in the exercises are based on direct application of rules and formulae. For example:

> Find the angle between the straight lines $y - x\sqrt{3} - 5 = 0$ and $y\sqrt{3} - x + 6 = 0$.

The NCF 2005 also suggests that mathematical modelling be introduced at this level making it possible to include the applications of some of the mathematical concepts that students are learning at this stage. Increasingly, many countries across the world, such as USA and UK, have witnessed a growing collection of didactical research on including mathematical applications and modelling at the high school level and this has impacted the mathematics curricula in those countries. Studies have shown that the benefits of including mathematical modelling and applications in the curriculum are manifold. They can:

- Help relate topics taught in the classroom to situations outside the classroom.
- Highlight the relevance of mathematics as a discipline.
- Focus on applications which help to build the students' interest in the subject.
- Offer direction in career options.

The textbooks: An overview

In this section we provide an overview of the topics in the senior secondary curriculum and the way the topics have been dealt with in the NCERT textbooks. For our study we have considered the NCERT mathematics textbooks of class XII of three representative years (1989, 2000 and 2007) and the NCERT mathematics textbooks of class XI of four representative years (1988, 1995, 2002 and 2006). We start with Class XII.

Class XII textbooks

The following topics have been a part of the syllabus through the years:

- Functions, limits and continuity
- Matrices and Determinants

- Continuity and Differentiability
- Application of Derivatives
- Integrals
- Applications of Integrals
- Differential Equations
- Vector Algebra
- Three Dimensional Geometry
- Probability

Chapters on Mathematical Logic, Correlation and Regression, Computing and Numerical Methods were part of the syllabus till 1989 and then removed. The chapter on Mathematical Logic included subtopics on mathematical statements and truth values, the use of Venn diagrams in logic, conjunction, disjunction, conditional statements, biconditional statements, truth tables and applications to switching circuits. These topics were reintroduced in 2003 in a chapter called 'Boolean Algebra' which included Boolean algebra as an algebraic structure, principle of duality, concepts of conjunction, disjunction, conditional statements, biconditional statements followed by truth tables and applications to switching circuits. In 2005 Boolean algebra was removed from the syllabus. The chapter on Numerical Methods dealt with basic numerical analysis and included methods for approximating the solutions of polynomial equations using successive bisection and the Newton-Raphson method. For solutions of systems of equations, the Gauss elimination method and the Gauss-Seidel iterative method were discussed.

The NCERT textbooks were revised in year 2000. The syllabus was divided into parts A, B and C. Part A (70 marks) was compulsory for all students, part B (30 marks) was for students of the science stream, and Part C (30 marks) for students of the commerce stream. Calculus accounted for nearly 50% of the syllabus and was included in the compulsory part. Part A also included the following topics: Matrices and Determinants, Probability, and Boolean Algebra. Part B included Vectors and Three-Dimensional Geometry. Part C included topics related to Commercial Mathematics (Partnership, Bills of Exchange) and Linear Programming.

In 2005 the textbooks were again revised based on the recommendations of the NCF 2005. The revised textbooks appeared in 2007. The three parts A, B and C were removed, and the topics were now sequenced in the following manner

- Relations and Functions
- Inverse Trigonometric Functions

- Matrices
- Determinants
- Continuity and Differentiability
- Application of Derivatives
- Integrals
- Applications of Integrals
- Differential Equations
- Vector Algebra
- Three Dimensional Geometry
- Linear Programming
- Probability

There was however no major change in the approach to dealing with these topics in the revised textbooks. For example, if we look at the chapter on Application of Derivatives we find the book of 2007 had problems and exercises similar to that of the previous years. In the section on Maxima and Minima the following problems have been appearing through the years:

> Find two positive numbers x and y such that their sum is 35 and the product x^2y^2 is a maximum.
>
> A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without a top, by cutting off squares from each corner and folding up the flaps. What should be the side of the squares to be cut off so that the volume of the box is a maximum?

In all the other chapters the content has largely remained the same in terms of concepts, explanations, solved examples and exercises.

From the year 2007, calculus was introduced in class XI. Thus the topic of limits was included in the textbook for class XI. Based on the recommendations of the NCF 2005 position paper, two chapters were added in the Appendix of the textbook for class XII.

> **Proofs in Mathematics:** This dealt with various types of proofs in mathematics, namely the direct and the indirect approach. In the direct approach, straightforward proof, mathematical induction and proof by exhaustion were discussed whereas in the indirect approach, proof by contradiction, proof by proving the contrapositive statement and proof by counter examples were discussed.
>
> **Mathematical Modelling:** This chapter highlighted the need and importance of mathematical modelling, the principles of modelling and steps involved in the

modelling process. It included examples from the topic of matrices, trigonometry and linear programming and ended with a paragraph on the limitations of the modelling process.

Class XI textbooks

Now we study the NCERT mathematics textbooks of class XI of four representative years (1988, 1995, 2002 and 2006). The following topics have remained in the syllabus through the years:

- Sets
- Relations and Functions (till 1995 this was combined with the chapter on sets)
- Trigonometric Functions
- Principle of Mathematical Induction
- Complex Numbers
- Quadratic equations
- Linear Inequalities
- Permutations and Combinations
- Binomial Theorem
- Sequences and Series
- Straight lines
- Conic Sections
- Statistics

Chapters on the topics Solution of Triangles, Inverse Trigonometric Functions, Linear Programming and Algorithms and Flowcharts were part of the syllabus till 1995 and removed later.

When the textbooks were revised in 2000, the class XI syllabus was divided into parts A, B and C just as was done with the syllabus of class XII. Part A (70 marks) was compulsory for all students, Part B (30 marks) was for students of the science stream, and Part C (30 marks) was for students of the commerce stream. Part A included all the topics mentioned above. A chapter on Mathematical Logic was added to this section. This included mathematical statements, logical connectives, truth tables, tautologies, logical equivalence, duality, algebra of statements and use of Venn diagrams in logic. Part B (for science students) included introductory chapters on Vector Algebra and Three Dimensional Geometry. These topics were dealt with in greater depth in Section B of the class XII textbook. Part C (for commerce students) included topics on Stocks, Shares and Debentures, Average and Partition Values and Index Numbers.

In 2005 the textbooks were again revised based on the recommendations of the NCF 2005, just as was done for Class XII. The revised textbooks appeared in 2007. The three parts A, B and C were removed, and the topics were now sequenced in the following manner:

- Sets
- Relations and Functions
- Trigonometric Functions (this included trigonometric equations but the subtopic on solution of triangles was removed)
- Mathematical Induction
- Complex Numbers and Quadratic Equations
- Linear Inequalities
- Permutations and Combinations
- Binomial Theorem
- Sequences and Series
- Coordinate Geometry (Straight lines, circles and conic sections)
- Introduction to Three Dimensional Geometry
- Mathematical Reasoning
- Statistics and Probability
- Limits and Derivatives (Calculus for the first time was introduced in class XI)

Two chapters were added in the appendix of the book.

Infinite Series: This included the subtopics on Binomial theorem for any index, Infinite geometric series. The topic of Exponential and Logarithmic series was moved to this chapter.

Mathematical Modelling: This chapter dealt with the concept of mathematical modelling which was extended in the class XII textbook.

Over the years there has not been any major change in the approach of dealing with the topics in terms of introducing or explaining the concepts or in the examples and exercises. For example, if we look at the chapter on Straight Lines we find that the subtopics covered are the same from 1988 to the present. These include equation of a straight line parallel to the axes, slope-point form of the equation of a line, two-point form, slope-intercept form, normal form, symmetric form, angle between two lines, condition of concurrency of three straight lines and translation of axes.

Also many of the questions in the exercises are based on direct application of the formulae and results presented in the chapters and have been repeated in the textbooks through the years. They appear to focus more on testing the student's manipulative skills. The following are some sample questions:

> Find the equation of the line perpendicular to $3x + 2y = 8$ and passing through the midpoint of the line joining $(5, -2)$ and $(2, 2)$.
>
> Find the angle between the straight lines $y - x\sqrt{3} - 5 = 0$ and $y\sqrt{3} - x + 6 = 0$.

Even the topic on Conic Sections has remained the same through the years. A diagram shows the sections of a right circular cone. This is followed by definitions, derivations of the equations for different conics and their properties. The problems and exercises are also based on the application of rules and formulae.

Missed opportunities in the Senior Secondary Curriculum

In this context we briefly comment on the "missed opportunities" in the syllabus. We ask, are there opportunities in the syllabus where without any further additions being made to the syllabus, much more can be done, where connections can be shown in a natural way, where technology enabled exploration is possible by the very nature of the topic? All this is asked keeping in mind the central tenet of the NCF 2005 document: "There is one main goal of mathematics education in school: the mathematisation of the child's thought processes." As noted by David Wheeler, "It is more useful to know how to mathematize than to know a lot of mathematics." When looked at in this light, numerous missed opportunities become visible; one finds numerous activities are possible which serve to unify different strands within the syllabus. The value of such activities is very great, because one of the traditional areas of weakness in our curriculum is the lack of attention given to connections between topics (and still less attention is given to connections across subjects). We quote one such example: the justification for the formula for volume of a pyramid: "V equals one-third (area of base) × (height)." This formula is made known to children when they are in the 9th or 10th standard and the presence of the factor 1/3 remains a point of mystery which never gets resolved. But if this topic is revisited when the students are in the 11th standard and have learnt a certain amount about finding a formula for the sum of the squares of the first n natural numbers, about proof by induction, and about limits, then this sets the stage for an illuminating activity in which we obtain the formula for volume by a graded sequence of steps, starting with dividing the pyramid into *n* slices of equal thickness parallel to the base (where *n* is large); finding the volume of each slice by treating it as a cuboid (some input is needed from the geometry of similar triangles here); summing the volumes (this is where the formula for $1^2 + 2^2 + ... + n^2$ is needed); estimating the limit computationally using a computer; and then actually determining the limit analytically. Finally one gets the known formula, and it is indeed a pleasure to see it emerge in front of our eyes. Finally one has the opportunity for demonstrating the correctness of the formula by an activity in the mathematics laboratory, in which we show how 6 congruent right pyramids of suitable size can fit together to yield a cube.

Given the value of shifting focus in the curriculum from content to process, it is important that we identify as many such opportunities as possible, because they bring many strands together and have great value in integrating concepts in a student's mind.

Assessment

Assessment in the Indian school education system is largely limited to the summative variety, and it is for the most part a device to measure cumulative learning: a device used to help teachers write reports and to help make pass/fail decisions. Thus there is little or no feedback into the learning process.

In few countries is it as true as in India that summative assessment in the form of a school leaving examination holds the key to one's future, in the sense of opening or closing doors of opportunity. The problem is of sufficient gravity that every single year there are suicides associated with it: children unable to cope with the disappointment and shame of failure, or with the fear of condemnation. Inevitably, the spectre of such assessment exercises a significant influence on the ambient educational culture, inviting poor educational practices and the creation of a powerful parallel education system called 'coaching'. Indeed, it invites criminal activity as well, through the leakage and sale of examination papers. It will be clear from these remarks that the school leaving examination is an extremely high stakes event.

School education in India follows a ten plus two system: ten years of compulsory schooling in which all students follow the same stream, followed by two years in which one chooses a set of optional subjects. These are grouped into streams: Mathematics, Physics, Chemistry (commonly known as 'MPC' or 'PCM'), Biology, Physics, Chemistry ('BiPC' or 'PCB') and so on.

The Central Board of Secondary Education (CBSE) and Council for the Indian School Certificate Examinations (CISCE) are national examination boards, and the better known schools in the country are associated with one or the other of these. CBSE follows the syllabus set by NCERT and uses NCERT textbooks, whereas CISCE sets its own syllabus, at both the 10th standard and 12th standard levels, and does not prescribe textbooks; schools are free to use textbooks of their choice. The academic standards of the two boards are comparable. The major difference is that CBSE has done away with the 10th standard examination, and has substituted it with the CCE system mentioned below. However the 12th standard examination continues in its original form.

Recently, the Central Board for Secondary Education has taken steps to bring in alternate assessment systems and has introduced a 'Continuous Comprehensive Evaluation' system (CCE for short; Continuous and Comprehensive Evaluation, 2011). It has prepared elaborate manuals for teachers on how CCE is to be transacted, and has held workshops on CCE methodology. The scheme certainly holds promise, but its long term effect on the academic culture of our schools remains to be seen.

Entry into colleges is decided either on the basis of the marks secured in the 12th standard or entrance examinations conducted by the respective colleges. Population pressures mean that entrances are a highly competitive process, particularly for prestigious colleges like the IITs (Indian Institutes of Technology) or AIIMS (All India Institute of Medical Sciences). This single fact has had a great influence on secondary school education — unfortunately, not a positive one; indeed, one that trickles down to the primary level. The entrance examinations of a few institutes have now become bench marks in the country. We shall look at the style of a few of these examinations later in this essay.

A situation peculiar to this country is the phenomenon of tutorial colleges ('coaching centres') which seek to prepare students for entrance to highly sought-after institutions. Some of these colleges are themselves highly sought after, and they have their own selection examinations, a situation which invites the possibility of an infinite iterative loop! One could laugh in good humour at the situation if it were not so wasteful of human energy. The methods used by these colleges amount to all-out drill, mastery of pattern recognition through analysis of past papers (a kind of reverse engineering set in an educational context), and reliance on huge memory banks. Over the last several decades these practices have gotten absorbed into the ambient educational culture of the country.

Comments from NCF 2005

The NCF 2005 document lists four core areas of concern: "(1) a sense of fear and failure regarding mathematics among a majority of children, (2) a curriculum that disappoints both a talented minority and a non-participating majority, (3) crude methods of assessment that encourage a perception of mathematics as mechanical computation, (4) lack of teacher preparation and support in the teaching of mathematics." It amplifies on the third point: "While what happens in class may alienate, it never evokes panic, as does the examination. Most of the problems cited … relate to the tyranny of procedure and memorization of formulas …, and the central reason for the ascendancy of procedure is the nature of assessment …. Tests are designed … to assess knowledge of procedure and memory of formulas. … Concept learning is replaced by procedural memory. Such antiquated and crude methods of assessment have to be thoroughly overhauled ….” It recommends the following: (1) Shift the focus of mathematics education from achieving narrow goals to higher goals; (2) Engage every student with a sense of success, and at the same time offer challenges to the emerging mathematician; (3) Change modes of assessment to examine mathematisation abilities rather than procedural knowledge; (4) Enrich teachers with a variety of mathematical resources.

The third and fourth points are of relevance here. With regard to the third point it adds: “Since the Board examination for Class X is for a certificate given by the State, implications of certified failure must be considered seriously. Given the reality of the educational scenario, the fact that Class X is a terminal point for many is relevant; applying the same … standard of assessment for these students as well as for rendering eligibility for the higher secondary stage seems indefensible. … [Given] the high failure rate in mathematics, we suggest that the Board examinations be restructured. They must ensure that all numerate citizens … become eligible for a State certificate. … Nearly half the content of the examination may be geared towards this. … However, the rest of the examination needs to challenge students far more than it does now, emphasizing competence and expertise rather than memory. Evaluating conceptual understanding rather than fast computational ability in the Board examinations will send a signal of intent to the entire system, and over a period of time, cause a shift in pedagogy as well. … These remarks pertain to all forms of summative examinations …. Multiple modes of assessment … need to be encouraged. This calls for … research and a wide variety of assessment models to be created and widely disseminated.” These are stirring words, and we hope they will be visited repeatedly by examination boards and state education departments in the years to come.

How examination boards handle assessment

In this section we study the way the CISCE deals with some selected topics and with problem solving in general.

Public examinations, high school level (class 10 – ICSE 2010)

(VAT computation) A manufacturer marks an article for Rs 5000. He sells it to a wholesaler at a discount of 25% on the marked price and the wholesaler sells it to a retailer at a discount of 15% on the marked price. The retailer sells it to a consumer at the marked price; at each stage the VAT is 8%. Calculate the VAT received by the Government from: (a) the wholesaler, (b) the retailer. [Question 6, ICSE 2010]

(Trigonometry) Without using trigonometric tables evaluate $\frac{\sin 35^\circ \cos 55^\circ \ + \ \cos 35^\circ \sin 55^\circ}{\csc^2 10^\circ - \tan^2 80^\circ}$

[Question 3(b), ICSE 2010]

Public examinations, high school level (class 12 – ISC 2010)

In this section we look at examples from one of the class 12 school leaving examinations.

(Matrix algebra) If the matrix $\begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$ is singular, find the value of x. [Question 1(i), ISC 2010]

(Coordinate geometry in two dimensions) Show that the line $y = x + \sqrt{7}$ touches the hyperbola $9x^2 - 16y^2 = 144$. [Question 1(iii), ISC 2010]

(Differential calculus) Using a variable substitution, find the derivative of $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$ with respect to x. [Question 5(a), ISC 2010]

(Integral calculus) Evaluate the following integral: $\int \frac{2\sin 2\theta - \cos\theta}{6 - \cos^2\theta - 4\sin\theta}\, d\theta$. [Question 6(a), ISC 2010]

(Differential equations) Solve the differential equation $\csc^3 x\; dy - \csc y\; dx = 0$. [Question 1(x), ISC 2010]

(Boolean algebra) x, y, z represent three switches in an "ON" position, and x', y', z' represent the same three switches in an "OFF" position. Construct a switching circuit representing the polynomial $(x+y)(x'+z) + y(y'+z')$. Using the laws of Boolean algebra, show that the above polynomial is equivalent to $xz + y$, and construct an equivalent switching circuit. [Question 4(b), ISC 2010]

(Data analysis: Moving averages) Consider the following data.

Dates	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Sales	2	5	0	12	13	25	45	13	31	18	11	2	3	1

Calculate three-day moving averages and display these and the original figures on the same graph. [Question 15(b), ISC 2010]

(Mean value theorem) Using Rolle's theorem, find a point on the curve $y = \sin x + \cos x - 1, x \in \left[0, \frac{\pi}{2}\right]$, at which the tangent is parallel to the x-axis. [Question 3(a), ISC 2010]

(L'Hopital's rule for limits) Evaluate: $\lim_{x\to\pi/2}\left[x\tan x - \frac{\pi}{2}\sec x\right]$. [Question 1(iv), ISC 2010]

Note the 'all or nothing' nature of the questions. This is typical of most examinations in the country. Note also the question asked in Data Analysis. This area is very poorly represented in the Indian curriculum; the coverage is limited to computation of a few statistics, with little or no interpretation of the numbers. The 'numeracy' component of such courses is low. But a characteristic feature of most such examinations is the high level of manipulative ability.

Project work

Progress has been made regarding project work. Some examination boards have a component for investigatory project work, which students do under a teacher's supervision. The marks allocation is modest: 10% of the total.

An example of a shift in thinking at the national level is the award of the KVPY scholarship based on original work in science or mathematics. It is possible to get the scholarship through a regular examination mode, but the organizers recognize that this mode discriminates against certain kinds of students, and hence that a dual approach is needed at a national level.

Competitive examinations: Engineering colleges

Here we study some questions asked in post school entrance examinations. We limit ourselves to two such exams: the AIEEE or All India Engineering Entrance Examination, and the JEE-IIT or Joint Entrance Examination for the IITs. A glance of the questions reveals the high level of preparation needed to do well in the exams. The time availability needs to be kept in mind: no more than three minutes for a typical multiple choice question! The situation is complicated by the fierce competition and very large number of candidates, which imply that a difference of just one mark may account for many hundreds of candidates. In response, students use strategies based on pattern recognition and memorization of large numbers of solved problems. One can well imagine the effects of this kind of high intensity input when it is continued for a year or longer: the effects on conceptual understanding, and on the psyche of individuals.

Here are the paper formats: ('PCM' is short for 'Physics, Chemistry, and Mathematics'):

- AIEEE: 30 MCQs each in PCM (duration 3 hours).
- JEE: Paper I: 28 questions each in PCM; 8 MCQ, single correct choice; 5 MCQ, one or more correct choices; 5 comprehension type MCQs; 10 numerical answer (with a two digit answer); 84 questions in all (duration 3 hours).
- JEE Paper II: 19 questions each in PCM; 6 MCQs, single correct choice; 5 numerical answer (with a single digit answer); 6 comprehension type MCQs; 2 matrix column matching type; 57 questions in all (duration 3 hours).

We take a look at some problems from AIEEE and JEE papers.

(Combinatorics): The number of 3×3 non-singular matrices with four entries as 1 and all other entries 0 is: (a) 5 (b) 6 (c) at least 7 (d) less than 4. [Question 71, AIEEE 2010]

(Coordinate geometry):

Statement 1: The point $A(3,1,6)$ is the mirror image of the point $B(1,3,4)$ in the plane $x-y+z=5$.

Statement 2: The plane $x-y+z=5$ bisects the line segment joining $A(3,1,6)$ and $B(1,3,4)$.

Then: (a) Statement 1 is true, statement 2 is false; (b) Statement 2 is true, statement 1 is false; (c) Statement 1 is true, statement 2 is true, and statement 2 is a correct explanation of statement 1; (d) Statement 1 is true, statement 2 is true, and statement 2 is not a correct explanation of statement 1. [Question 73, AIEEE 2010]

(Probability and combinatorics): Four numbers are chosen at random (without replacement) from the set $\{1,2,3,...,20\}$.

Statement 1: The probability that the four numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement 2: If the four chosen numbers form an AP, then the set of all possible common differences is $\{\pm1,\pm2,\pm3,\pm4,\pm5\}$.

Then: (a) Statement 1 is true, statement 2 is false; (b) Statement 2 is true, statement 1 is false; (c) Statement 1 is true, statement 2 is true, and statement 2 is a correct explanation of statement 1; (d) Statement 1 is true, statement 2 is true, and statement 2 is not a correct explanation of statement 1. [Question 72, AIEEE 2010]

(Trigonometry): Let P and Q denote the statements

P: $cosA+cosB+cosC=0$

Q: $sinA+sinB+sinC=0$

If $cos(B-C)+cos(C-A)+cos(A-B)=-\frac{3}{2}$ then:

(a) P is true and Q is false (b) P is false and Q is true (c) both P and Q are true (d) both P and Q are false. [Question 64, AIEEE 2009]

(Calculus, derivatives): Given $P(x)=x^4+ax^3+bx^2+cx+d$ such that $x=0$ is the only root of $P'(x)=0$. If $P(-1)<P(1)$ then in the interval $[-1,1]$: **(a)** $P(-1)$ is the minimum and $P(1)$ is the maximum of P; **(b)** $P(-1)$ is not the minimum but $P(1)$ is the maximum of P; **(c)** $P(-1)$ is the minimum and $P(1)$ is not the maximum of P, **(d)** neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P. [Question 84, AIEEE 2009]

(Complex numbers): Let $z=x+iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3+\bar{z}z^3=350$ is: (a) 48 (b) 32 (c) 40 (d) 80. [Question 24, JEE 2009]

(Trigonometry): In triangle ABC with fixed base BC, the vertex A moves so that $cosB + cosC = 4sin^2\frac{A}{2}$. If a, b, c denote the lengths of the sides opposite to the angles A, B, C then: (i) $b + c^2 = 4a$ (ii) $b + c = 2a$ (iii) the locus of point A is an ellipse (iv) the locus of point A is a pair of straight lines. [Question 31, JEE 2009; this is a MCQ in which there is more than one correct choice]

(Integral calculus): Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - f'(t)^2}\, dt = \int_0^x f(t)dt$ for $0 \leq x \leq 1$, and $f(0) = 0$, then: **(a)** $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$; **(b)** $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$; **(c)** $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$; **(d)** $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$. [Question 23, JEE 2009]

(Vectors): If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right) . \left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$ then: (i) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar; (ii) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar; (iii) $\vec{b}, \vec{d}$ are non-parallel; (iv) $\vec{a}, \vec{d}$ are parallel, and $\vec{b}, \vec{c}$ are parallel. [Question 26, JEE 2009]

(Calculus, limits): Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - x^4/4}{x^4}$ where $a > 0$; then: (i) $a = 2$ (ii) $a = 1$ (iii) $L = \frac{1}{64}$ (iv) $L = \frac{1}{32}$. [Question 30, JEE 2009]

Trick questions are also asked in such examinations: those for which one can get the answer through a trick peculiar to just that question, or those in which one can examine the multiple options given and quickly eliminate all but one of them, typically by some very elementary argument which barely does justice to the depth of the question itself.

The relevant question yet again lies in the social setting that lies behind these entrance examinations–the coaching culture, which is so numbing of human initiative, and the intense expectations placed on pupils by parents. The really sad part lies in the intention: the primary purpose behind these highly sophisticated examinations is, surely, to filter out and exclude, rather than to include and nurture.

Educational assessment is an exceedingly complex matter, for the issues it touches are so many, and of so varied a nature: from educational pedagogy to inherited societal problems such as caste barriers with which India is struggling. (These have held back the movement in society of whole communities within the country.) Yet, it is not an open problem in the sense that a mathematical problem may be described as 'open'. The primary difficulty, we feel, is a lack of clarity of educational vision. What is needed is not so much the subtlety of mathematical questions (not that this is not relevant, but it is far from being a primary issue) but the matter of quality of nurture and care that should inform and be the ultimate driving force behind assessment.

The question of technology

Mathematics has for years been the common language for classification, representation and analysis. Learning mathematics forms an integral part of a child's education. Yet, it is also the subject traditionally perceived as difficult. The primary reason for this is the significant gap between content and pedagogy. The last few decades have witnessed serious experimentation and research in mathematics education all over the world and there has been a shift of paradigm as far as mathematics teaching-learning is concerned. Mathematics education is being revolutionized with the advent of new and powerful technological tools. Because of these tools mathematics education can focus on problem solving and reasoning that empower students to explore, conjecture and reason logically. While traditional mathematics is often fraught with rote memorization of procedures, computational algorithms, paper-pencil-drills and manipulation of symbols, the use of technology encourages teachers and students to engage in deep mathematical thinking involving analysis, problem posing, problem solving and rich conceptual understanding.

Many countries are increasingly using technology in mathematics teaching and learning but this is not the case in India. Although integration of technology in schools is not uncommon in India, its use in mathematics teaching and learning in this country is not prevalent. In this section we list the challenges facing mathematics education in India and suggest ways in which technology can play a role to overcome these challenges.

The challenges facing school mathematics education in India may be broadly categorized under the following heads.

- Transaction of curriculum
- Inappropriate assessment
- Teacher preparation

Each of these can be significantly impacted by the appropriate use of technology. For example, technology can aid in the visualization of concepts, in exploration and discovery, in bringing the experimental approach into mathematics, in focusing on applications, in redefining the teacher's role, in helping sustain students' interest, in individualized grading and assessment, and in teacher outreach.

But while technology has profound implications for teaching and learning, it does not by itself supply solutions. The mere provision of technology in a class does not solve the problems faced in teaching. It is thus essential that serious research and experimentation go into the use of technology as an aid to teaching mathematics. Mathematics educators must consider in depth the possibilities created by computer software and handheld calculators.

Implementation of technology poses many challenges, the greatest being the socio-economic challenge. The priority of Government is to reach education to the masses. Technology must be cost effective and easy to deploy. The last few years have witnessed extensive use of computer technology in schools. However mathematics teaching continues in the traditional 'chalk and board' manner. Technology, if used for teaching mathematics, is primarily for demonstration purposes and does not involve the student actively. It is imperative that a mathematics curriculum be designed which integrates technology.

To successfully face the challenges in implementing technology in the Indian context, pre-service teacher education programmes must be designed where student teachers are taught mathematics using various technological tools. This will help develop new perspectives on integrating technology in their teaching-learning. In-service teacher training programmes must focus on changing teachers' mindset towards technology and helping them overcome their 'technological anxiety'. Technology must play a role in developing their pedagogical content knowledge. These professional development programmes must be held in a sustainable manner. This requires collaboration with technology solution providers who can provide ongoing support for the use of the technological tools in the schools. Students should be given adequate access to technology on a daily basis. Further, involvement of teachers on a large scale will require fundamental changes in teaching practices.

Mathematics laboratories are a medium through which students can explore and visualize mathematical concepts and ideas through the use of technology, and the potentialities of this medium need to be explored to its fullest extent.

In conclusion one may say that much work remains to be done if we are to effectively use the power and reach of modern technology in mathematics education in India. Perhaps the area of greatest challenge is teacher preparation: developing sustainable professional development programmes for teachers which not only enhance the skills of the teacher in terms of usage of various technological tools but also focus on improving their pedagogical content knowledge using technology. Another challenge is that the present curriculum does not readily lend itself to integration of technology. The goals of mathematics learning and assessment need to undergo a major shift in paradigm in a technology integrated mathematics curriculum. Also technology must be cost effective and easy to deploy in order to achieve large scale integration in schools and teacher education institutions. All this has tremendous implications in terms of infrastructure requirements. So a great deal of work remains to be done, but the benefits would clearly be enormous.

Concluding note: Some reflections

1. As mentioned earlier the content has largely remained the same over the years. The approach to dealing with the topics has also remained the same – to a much greater degree. Every chapter usually begins with a brief introductory note which sometimes includes a historical background of the development of the field, and then introduces the basic concepts of the topic. The chapter is divided into sections and sub-sections which deal with definitions, theorems, results, examples and exercises. This has been the format in which the topics have been written over the years.

2. However given the fact that the demands of mathematics education are changing, this needs to reflect in the mathematics curriculum and also in the way the topics are dealt with in the textbooks. The use of real world applications and modelling in dealing with concepts in various topics will help create a context for applying mathematical theory; this can act as a motivating factor. It will help highlight the beauty and relevance of mathematics as a discipline and at the same time focus on the usefulness of the topics being discussed. Indeed if the higher secondary stage is the 'launching pad' from which the student is guided towards career choices, the mathematics curriculum needs to provide inputs in terms of application to various fields of study such as the engineering sciences, biological sciences, economics and econometrics and even the social sciences.
   If applications have to be introduced, usefulness cannot be the only criteria for including a particular topic. The structure of mathematics is also critical in deciding which applications should be introduced in a particular topic, and in what sequence. In fact the applications should blend into the chapter enabling the student to

   - Learn new mathematical content;
   - Learn that mathematics applies to real problems;
   - Apply or practice the mathematics they have learnt;
   - See that applications are important because they lead to new mathematical problems and hence to the development of mathematics.

   Clearly the applications and modelling approach could be used to develop the concepts in various topics such as calculus, matrices and determinants and probability. Including material on mathematical modelling in the Appendix of the textbook does not appear to serve much purpose. Rather, a chapter exclusively on mathematical modelling could be included, dealing with models related to traffic flow, cryptography, optimization, genetics and so on. This will help students see how mathematics applies to various fields of study.

3. The senior secondary mathematics curriculum needs to have adequate emphasis on the understanding of mathematics as well as problem solving skills. Presently the emphasis seems to be largely on the computational aspects. Topics which form the foundation of Pure Mathematics courses at the undergraduate level may be included in the curriculum at an elementary level such as Group Theory. Similarly topics in applied mathematics such as Graph Theory, Game Theory, Markov chains and Numerical Methods may be included in the curriculum at an elementary level, and students given a choice to opt from among these topics. This will help align the senior secondary mathematics curriculum with the requirements of the mathematics courses at the undergraduate level.

   It is apt to say that the senior secondary mathematics curriculum needs to undergo a major shift in terms of structure and presentation of content.

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