7. Curriculum and pedagogy in mathematics: Focus on the tertiary level

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Introduction

The aim of this essay is to present a critical overview of mathematics education at the tertiary level in India. 'Mathematics at the tertiary level' would typically refer to mathematics taught at the undergraduate and post-graduate levels and would also cover research degrees in mathematics. By focusing primarily on undergraduate mathematics education and by extrapolating from different experiences we hope to throw some light on curriculum and pedagogy in mathematics at the tertiary level in India.

Only a tiny fraction of the Indian population (1.2 billion strong) enters higher education – it is yet vast in numbers, and expanding rapidly. India is therefore faced with the triangle of quantity, quality and equality. The immediate twin challenges that the country faces are: how do we increase the percentage of the population that accesses higher education? And how do we improve the quality of higher education?

Undergraduate education is akin to a pivot or keystone that holds together myriad strands that contribute to society. It is the case that whether we consider a teacher at school or a lecturer at the University, a researcher or a manager, or anyone holding a white-collar job, the one common aspect they share regarding higher education is that of having had undergraduate education.

The degree at the undergraduate level paves the way for the future. It is usually carefully chosen keeping in mind interests, aptitude and career opportunities ahead. It comprises those years in one's life where one is moving from a system governed by 'restrictions' to one that is about 'making choices and decisions' that stay with the rest of one's life. It is also the first adult interaction that one has with education.

The French philosopher and mathematician René Descartes (1596-1650) said "Mathematics is a more powerful instrument of knowledge than any other that has been bequeathed to us by human agency." Mathematics is explicitly and implicitly present in many things of importance to society. Mathematics has a role to play in so many different fields: innovations in medicine, digital encryption, communication technology, modelling real life phenomena, predicting disasters, organisation of enterprises, business and transport to name a few.

Yet, in general there is not only a lack of awareness about the indispensability of mathematics but instead there is a marked tendency to 'ignore' mathematics. Indeed, it is fashionable to acknowledge publicly the deep-seated fear of the subject that stems from memories of bad experiences with mathematics usually encountered at school. It is important for mathematicians and mathematics educators to acknowledge and seek ways of changing this. It is neither good for the discipline nor society to be in a situation where possibly over 50% can only recount dislike and unhappiness associated with mathematics.

At the heart of mathematics education lies undergraduate mathematics education. It would be impossible to tackle any of the problems associated with mathematics education, at any level without intervention at the undergraduate level. After all, the harbingers of change, if there are to be any, will be the teachers, policy makers, the creators and imparters of curriculum and pedagogy. And each one of them will have been shaped by their undergraduate (mathematics) education. Hence it is necessary that we examine the doctrines that govern undergraduate mathematics education in India.

What institutions or courses comprise undergraduate mathematics education? What should be the aims and goals of undergraduate mathematics education? What is the state of undergraduate mathematics education in our country? Are our courses geared to meeting the stated goals and aims? These are some of the questions that we shall cover in the essay. We will also touch briefly on mathematics education at post-graduate and higher levels by looking at programmes and initiatives in India that try to strengthen mathematics at the tertiary level.

Undergraduate mathematics education

Before examining the different types of degrees under which mathematics is taught at the undergraduate level we should dwell on possible goals of undergraduate mathematics education. One would expect a student leaving with an undergraduate degree to have some knowledge about the society she lives in and to have a set of skills that include the ability to communicate, to work in a team and to use modern tools like computers and computer networks. This should be enhanced by mathematical knowledge and skills gained through courses done, and should result in the ability to apply mathematical techniques to analyse, model and solve problems.

A very important goal of undergraduate mathematics education is also to create a pool of students who would continue with studying mathematics at the postgraduate and research levels and consequently also enlarge the pool of people who will form the educators of the next generation.

Are these goals being met? If these goals were met then we would be creating not only a good pool of future mathematicians but also meet the demands of the public and private sectors, business and society. A general statement made by the corporate sector a few years ago was that only a fourth of graduates in India were employable. This is probably true for Mathematics graduates too. A critical review of curriculum and pedagogy at the undergraduate level is therefore essential.

From the past to the present

Mathematics is taught as part of many different undergraduate degrees in India. Mathematics education at the undergraduate level needs to therefore cover the many different programmes in science, engineering, commerce and social sciences. Understandably, the requirements of knowledge of mathematics vary considerably for different programmes and the curricula for respective programmes are framed accordingly.

The most intensive mathematics degree at the undergraduate level is the three or (four) year Honours programme in Mathematics. Typically, in such programmes two-thirds of the courses are from the mathematics discipline and a third would be from disciplines other than mathematics.

Questions that are related and must be explored given the interdisciplinary and multidisciplinary needs of society are: Is it possible to create and run multidisciplinary programmes that are equally intensive with mathematics as one, but not the only, focus of study? Is it possible for mathematics to be part of undergraduate programmes that are not focused on mathematics without such courses being entirely oriented towards utilitarian skills?

The next category consists of the Bachelors in Arts or Bachelors in Science Programmes that offer mathematics as one of two or three disciplines that a student has to study. The weightage of mathematics courses in these programmes would vary from a third to half. Students pursuing physics, economics or four-year engineering degrees at the undergraduate level would also do a fair number of mathematics courses. Mostly, the faculty involved in teaching these courses would be from mathematics departments. Students pursuing undergraduate degrees in 'commerce' (involving courses like commerce, accountancy, etc.) also have some courses in mathematics.

So what was the mathematics curriculum at the undergraduate level like about half a century or so ago: What were the books used by undergraduate students of mathematics? How many undergraduates with a mathematics background were produced then in India?

What kind of careers did they choose? Many of the topics that were taught then would be termed as classical and are no longer part of current curriculum. When did undergraduate mathematics curriculum reform take place in India? How were faculty members trained and prepared to handle the new changes? Particularly, what were the changes that paved the way for Analysis and Abstract Algebra to enter the curriculum and replace courses like astronomy or tensor calculus? We explore answers to these questions based on written replies to questions sent to some senior mathematicians.

Professor M. S. Raghunthan[1] is one of India's pre-eminent mathematicians. We posed questions to him over e-mail on his undergraduate education.

Reproduced below are the questions and answers of Raghunathan (MSR) about his undergraduate education. Some footnotes have been added to clarify and give more information.

Question: In which institution did you do your undergraduate degree and when?

MSR: I got my degree in 1960. I was a student of Vivekananda College in Madras.

Question: What did the undergraduate mathematics curriculum consist of then?

MSR: There was considerable variation in the curriculum in different universities especially at the MSc level. Here is roughly what I was taught in the BA (hons) course of the Madras University. In those days Maths was clubbed with the Arts in Madras. The "honours" course in the Southern Universities were different from those in the North. It was a 3-year course after "Intermediate"[2] while BA was a 2-year course. At the end of the course the Honours students wrote the same exams as the MA students but were awarded the BA Honours degree. It could be converted into an MA[3] a year later by paying a fee! The certificate actually said that the candidate is awarded the MA "by efflux of time"!

In the first year we were taught Differential Calculus a la Joseph Edwards[4]

[1] See http://www.math.tifr.res.in/~dani/msrfr.pdf for more on Professor Raghunathan.

[2] During that period in the South and possibly elsewhere in India, students spent 11 years at school followed by 2 years of 'intermediate' study at a college. The BA (Honours) described above was a three-year programme after intermediate and resulted in the successful candidates being able to convert their BA (Honours) to an MA degree. The BA was a 2-year degree and students wishing to pursue a MA after the BA would have classes with the second year cohort of the of the 3-year BA (Honours).

[3] This is in keeping with the tradition followed at Oxford and Cambridge. Even now at Oxford or Cambridge there are no taught MA courses. Instead a BA Honours can be converted to an MA after paying a fee and after waiting for several terms. This stems from an old tradition in 'Arts' where after a BA you were apprenticed to a Master with whom you honed your skills and hence reaching a new status of 'Master of arts' after a suitable period.

[4] Joseph Edwards, *Differential calculus: with applications and numerous examples*; *An elementary treatise*, Macmillan 1886.

(Ramanujan[5] studied this book; it was no rigorous treatment of the calculus - expanded functions happily in power series without talking about convergence even while different forms of "reminder after *n* terms" were given. It had also some differential geometry of curves in the plane. There was some Trigonometry (the syllabus was the contents of an "advanced" text book by S. L. Loney[6]). There was some algebra - Barnard and Child[7] covered all that we studied but had more. Then there was some Euclidean Geometry which went well beyond Euclid - things like Brocard[8] points, the kind of stuff, later generations do not know much about. In the second year we were introduced to Analysis (now things got rigorous) - Shantinarayan's[9] book essentially covered the syllabus. Hardy's[10] pure mathematics was also used. Then there were (particle) Dynamics and Statics courses through the second year. Loney's[11] advanced level books were used. There was a course on synthetic projective geometry (conic sections), but there was also "analytic geometry" - here the material covered is in the book by Askwith[12] - basically again all about conics, but dealt with through equations. In the final year we had solid geometry - that is three-dimensional "analytic" geometry, more analysis, rigid dynamics (Loney's book again) and some sophisticated stuff like Lagrange's equations and Hamilton's action principle, etc. There was a paper on "astronomy" which was essentially spherical trigonometry. There were "two special" papers which varied from college to college. Our college offered Hydrodynamics and Complex Analysis. There were other colleges that offered "Modern Algebra" in place of Hydrodynamics.

The BA "pass" course offered some of the above material eschewing anything rigorous! S L Loney had textbooks, which were not "advanced" in all the subjects mentioned above and that defined the curriculum for the BA.

In the North, there were places where (local) differential (which really amounted to tensor calculus) was taught and there were other subjects

[5] See http://www-history.mcs.st-and.ac.uk/Biographies/Ramanujan.html for more on Ramanujan.

[6] S L Loney, *Plane Trigonometry*, Cambridge: At the University Press, 1893 (reprinted now by Michigan Historical Reprint Series)

[7] Barnard S and Child J M, *Higher Algebra*, Macmillan and Co Limited, London 1939.

[8] See http://mathworld.wolfram.com/BrocardPoints.html.

[9] Shanti Narayan, *A course in Mathematical Analysis*, first published in 1949. (A revised edition of this book is now publsished by S. Chand and Co, Delhi, India.)

[10] G H Hardy, *A course in pure mathematics*, first published in 1908 by Cambridge University Press. (Latest Reprint in 2002.)

[11] S L Loney, *Elements of Statics and Dynamics*, Cambridge: At the University Press, 1932.

[12] E H Askwith, *The analytical geometry of conic sections*, *A. and C.* Black, 1908.

like relativity and Electromagnetism that were also offered at the master's level.

Question: How many mathematics graduates would India have produced each year in that era?

MSR: Madras University perhaps produced some 100 to 150 MAs and about 500 BAs - that is a guess but not entirely baseless. There were probably about 20 universities in the country awarding degrees in Mathematics and a dozen offering MA.

Question: What kind of careers did mathematics graduates pursue in that period?

MSR: Teaching of course. A good number from Madras ended up in the accountant general's office. Some became clerks and the bright ones wrote the IAS[13], IPS[14] or Central Services exams. Actuaries was another.

Question: If the mathematics curriculum was very different, when did topics like Abstract Algebra and Real Analysis become part of the undergraduate curriculum in India? Was there any special effort made to train teachers to teach these new subjects?

MSR: I think in the North Real analysis was being taught in the BA classes of some universities. Abstract algebra was taught only at the master's level – if at all – in those days, even in the North. Abstract algebra began to be taught perhaps in the mid seventies.

Professor S. G. Dani,[15] another of India's eminent mathematicians, responded to our similar questions and reported that he completed his graduation in 1967 with physics as his major and mathematics as a minor subject. During his time, mathematics curricula were very narrow. In particular, courses like group theory, complex analysis, linear algebra, basic differential geometry were not included in the undergraduate syllabus. Like Raghunathan, Dani recalls the books by Loney and Shantinarayan as popular ones in those days.

We also get a fair idea of curriculum reform undertaken in the state of Gujarat (located in Western India) through 'History of Curricular reforms in Mathematics' by Professor M. H. Vasavada[16]. Below are excerpts from his piece. This has been produced almost verbatim

[13] Indian Administrative Service

[14] Indian Police Service

[15] Please see http://www.math.tifr.res.in/~dani/ and http://aimconf.webs.com/profiles/S%20G%20Dani.pdf for more on Professor Dani.

[16] After passing the MSc examination, Professor Vasavada first joined V P Science College, Vallabh Vidyanagar, and then the Department of Mathematics, M S University of Baroda, as a lecturer. In 1964, he went to USA under the Fulbright travel grant and joined the University of Wisconsin for his PhD degree. After getting his PhD degree in Functional Analysis under L C Young, he returned to India as a Reader

except for a few minor corrections and a few footnotes that have been added.

> The revision in syllabus came first at PG level and then at UG level. The logic was that the new appointees in the colleges, who got their MSc degree with the revised syllabus, would have already learnt the new material. The lead for PG reforms in Gujarat was taken by M. S. University of Baroda[17]. Dr. U. N. Singh[18], who had a PhD from University of Allahabad and a DSc from Sorbonne (Paris, France), started teaching Measure Theory and Lebesgue integration to MSc students as far back as 1958. Some teachers were also sent to TIFR[19] to learn new subjects like abstract algebra. Then research students under Dr Singh started working in modern branches like operator theory. Also some of the research workers who had gone to USA in the early sixties for their PhD started returning and took up assignments in various university departments. This made the transition from old to new at PG level smooth. By 1970, the PG mathematics departments in universities in Gujarat had revised their syllabi and started teaching new courses.
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> The change at the UG level was slow in coming. There were a large number of colleges and a large number of college teachers. But the summer programmes and the in service programmes for college teachers were organised by university departments with funding from UGC and by 1975, the courses at UG level also were revised. The main subjects taught at UG in the old syllabus were Classical Algebra, Calculus of functions of one and several variables, Real Analysis, Pure Plane Geometry, Analytic Geometry of two and three dimensions, Differential Equations, Statics, Hydrostatics, Dynamics, Astronomy and Electricity and magnetism.

Vasavada's account gives us a detailed picture of the old Syllabus that was taught at both the undergraduate level and the postgraduate level before syllabus reforms came about in 1975 for the undergraduate level and 1970s for the postgraduate level. He attributes the coming of 'modern topics' to the efforts of Professor U. N. Singh.

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in M. S. University. In 1972, he joined the Postgraduate Department of Mathematics of the Sardar Patel University at Vallabh Vidyanagar as a Professor of Mathematics and the Head of the Department and retired in 1996.

[17] See http://www.msubaroda.ac.in/.

[18] "In January 1958, Professor U.N. Singh, D.Sc. (Paris), was appointed as the Professor and Head of the Department. With this advent, the Department became quite active and members were sent for visits/ Ph.D. programme abroad, several summer schools in Modern Mathematics was organized. In fact, the American Mathematical Monthly in an article in 1966 lauded the curriculum of our department as an ideal curriculum. Professor U.N. Singh left the Department for Delhi University in 1966." Excerpt from the write-up on the Department of Mathematics, M S University Baroda.

[19] See http://www.tifr.res.in/index.php/en/ for more on Tata Institute of Fundamental Research.

Undergraduate Level (Old Syllabi)

Algebra: Numbers, Symmetric functions, Theory of Equations (Cubic and Biqudratic equations), Summation of finite and infinite series, Determinants. Book: Higher Algebra by Barnard and Child.
Calculus: Leibnitz rule for nth derivative, Partial differentiation, Singular points, asymptotes, envelopes, curvature, curve tracing, maxima and minima for functions of several variables, reduction formulae for integrals
Books: Calculus by Edwards; Differential Calculus by Shantinarayan
Analysis: Continuous and differentiable functions of one and several variables, series of functions, uniform convergence, Fourier series, Riemann Integration, Mean Value Theorems of differentiation and integration, Double and Triple integrals, Trigonometric, exponential and logarithmic functions
Books: Pure Mathematics by G. H. Hardy, Analysis by E G Phillips, Real Analysis by G S Mahajani, Calculus and Analysis by G K Hebalkar.
Pure Geometry: Projection, cross ratios, Perspectives, Harmonic Section, Involution, Conics
Book: A course of Pure Geometry by E H Askwith
Analytic Geometry of Two Dimensions: Polar Coordinates, General Equation of Second Degree.
Analytic Geometry of Three Dimensions: Lines, planes, conicoids.
Books: Coordinate Solid Geometry – Elementary Treatise by R J T Bell, Analytical Solid Geometry by Shantinarayan.
Differential Equations: Linear differential equations of second and higher order, systems of linear differential equations.
Books: Differential Equations by Daniel Murray.
Statics, Hydrostatics and Dynamics:
Statics and Dynamics of a particle, Elements of Hydrostatics
Books in all the three subjects were by Ramsey
Astronomy
Book: Astronomy by Smart
Electricity and Magnetism

Postgraduate level

Subjects taught:
Algebra: Continued Fractions, Differential Equations, Congruence modulo a number, Fermat's theorem and its Euler's extension, Indeterminate equations, Infinite products.
Book: Higher Algebra by Barnard and Child
Coordinate geometry of three dimensions
Book: Coordinate Geometry of Three Dimensions by R J T Bell
Plane Geometry: Conics, Recirocation, Inversion
Book: Plane Geometry by E H Askwith
Spherical Trigonometry:
Book: Spherical Trigonometry by Todhunter and Leathem
Higher Plane Curves: The study of curves in the plane represented by cubics and higher degree equations
Real Analysis
Complex Analysis
Books: Complex Analysis by E.G.Phillips
Also there were two optional papers, to be chosen from:
Functions of a Complex variable
Statics and Dynamics

Astronomy
Differential Geometry

Professor Dinesh Singh[20], son of U. N. Singh, and himself a mathematician of repute recollected the following information regarding his father's undergraduate and postgraduate years of study. U. N. Singh did his undergraduate degree and an MA in mathematics at Allahabad University in the 1940s. In the MA, U. N. Singh was already using books by Copson[21], Titchmarsh[22] and Hobson[23]. Books on Abstract Algebra by I N Herstein (*Topics in Algebra*, John Wiley and Sons, 1964), Birkhoff and Maclane (*A Survey of modern Algebra*, A. K. Peters, 1977) were beginning to be used in the 60s and 70s respectively. Additionally, books on Analysis by Walter Rudin (*Principles of Mathematical Analysis*, McGraw Hill, 1953) and Royden (*Real Analysis*, Prentice-Hall, 1963) were also being used. The era of 'modern mathematics' seemed to have arrived or had it? The answer to this can only be given when we consider the present syllabi. We do this in the next section.

An analysis of current mathematics curriculum at the undergraduate level

The mathematics curriculum taught in various programmes is presented in this section. This covers a gamut of programmes from those that have over two-thirds of their coursework consisting of mathematics courses to those in which a third or less consists of mathematics courses. The focus will primarily be on high weightage mathematics programmes.

The BA (Bachelor of Arts) and BSc (Bachelor of Science) programmes are of three year duration and generally they are run by different universities. These programmes with Honours or Major in mathematics are aimed at training the student for higher (graduate) studies in mathematics. On the other hand, mathematics curriculum in BA/ BSc programmes without Honours in mathematics (including Honours in another subject) caters to the need of a wide range of courses in sciences and social sciences. Similar is the case for the mathematics curriculum for BCom (Bachelor of Commerce) programmes run by the universities. There are several institutions, which have five-year integrated MSc programmes in mathematics which are aimed to train the student either for higher studies in mathematics leading to research or for industry. The Indian Institutes

[20] See http://www.du.ac.in/fileadmin/DU/faculty/PDF/2906_02.pdf for more on Professor Dinesh Singh.

[21] See http://www-history.mcs.st-and.ac.uk/Biographies/Copson.html for more on Copson.

[22] See http://www-history.mcs.st-andrews.ac.uk/Biographies/Titchmarsh.html for more on Titchmarsh.

[23] See http://www-history.mcs.st-and.ac.uk/Biographies/Hobson.html for more on Hobson. Incidentally, E. W. Hobson was one of the mathematicians that Ramanujan wrote to in England seeking mathematical advice. This was before Ramanujan wrote his now famous letter to G. H. Hardy.

of Science Education and Research (IISER)[24], National Institute of Science Education and Research (NISER)[25] are examples of such institutions. The mathematics curriculum for these programmes is quite ambitious. This is also the case for the curriculum of the undergraduate programmes of institutes like the Chennai Mathematical Institute (CMI)[26] and the Indian Statistical Institutes (ISI)[27].

The mathematics courses in the undergraduate engineering programmes (of four years duration) conducted by a large number of engineering institutes/ colleges vary substantially across the institutions and according to the requirements of the programmes. For example, the mathematics courses in the programmes of the institutes like the Indian Institutes of Technology[28] and National institutes of Technology[29] are more rigorous than those of many of the state and private engineering colleges which are content with a working knowledge of mathematics.

Curriculum for BA, BSc, four-year BS and five-year Integrated MSc programmes in universities and institutes

Generally the Mathematics courses of both BSc and BA programmes (with Honours/ Major in Mathematics) are the same; the two programmes differ in the nature of the subjects a student chooses from in addition to mathematics, that is, whether from science or social sciences stream.

The BA/ BSc (Honours/ Major in Mathematics) curriculum of most of the universities include the following as compulsory courses:

1. Classical Algebra
Algebra of complex numbers, geometry in complex plane, de Moivre's Theorem and applications, roots of polynomials, Fundamental Theorem of Algebra (statement).
Theory of equations, relations between the roots and coefficients of polynomial equations in one variable, transformation of equations, Descarte's rule of signs, symmetric functions of roots, solution of cubic equation by Cardan's method.
Set, relations and functions, binary operations, Integers, division algorithm, Principle of Mathematical Induction, well ordering of positive integers, Fundamental Theorem of Arithmetic.

2. Linear Algebra
System of linear equations, real matrices, determinants and inverse of a matrix, row reduction and echelon form.
Vector spaces, linear span, linear dependence and independence of vectors, basis and dimension, quotient spaces and its dimension, rank nullity theorem, sums and direct sum of subspaces.
Linear transformations and their representation as matrices, the algebra of linear transformations, the rank nullity theorem, change of basis, dual spaces.

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[24] See http://en.wikipedia.org/wiki/Indian_Institutes_of_Science_Education_and_Research.
[25] See http://www.niser.ac.in/.
[26] See http://www.cmi.ac.in/.
[27] See http://www.isical.ac.in/.
[28] See http://en.wikipedia.org/wiki/Indian_Institutes_of_Technology.
[29] See http://en.wikipedia.org/wiki/National_Institutes_of_Technology.

Eigenvalues and eigenvectors, characteristic equation of a matrix, Cayley Hamilton theorem, minimal polynomial, characteristic and minimal polynomial of linear operators.

3. Calculus

Differential Calculus: Higher order derivatives, Leibniz rule, L'Hopital's rules.

Functions of several variables, level curves and surfaces, limits and continuity, first and higher order partial derivatives, tangent plane, directional derivatives and the gradient, extrema of functions of two variables, method of Lagrange multipliers.

Integral Calculus: Integration techniques, definite integrals, Improper Integrals, applications in finding areas, arc lengths and volumes of revolutions.

Double integral over rectangular and nonrectangular regions, triple integral, change of variables, divergence and curl, line integrals, Fundamental Theorem and path independence, Green's theorem, surface integrals, Stokes' theorem, Divergence theorem.

4.Differential Equations

Ordinary Differential Equations: First order equations, exact differential equations, integrating factors, Bernoulli equations, existence and uniqueness theorem, applications.

Higher-order linear differential equations, solutions of homogeneous and nonhomogeneous equations, method of variation of parameters, operator method; series solutions of linear differential equations, Legendre equation and Legendre polynomials, Bessel equation and Bessel functions of first and second kinds

Systems of first-order equations, phase plane, critical points, stability.

Partial Differential equations: First order partial differential equations; solutions of linear and nonlinear first order PDEs; classification of second-order PDEs; method of characteristics; boundary and initial value problems (Dirichlet and Neumann type) involving wave equation, heat conduction equation, Laplace's equations and solutions by method of separation of variables, initial boundary value problems in non-rectangular coordinates.

5. Analysis

Real line, field and order properties, Completeness property, Archimedean property, density of rationals, open and closed sets, closure, sequence and convergence, Cauchy's criterion, monotone convergence theorem, Bolzano-Weierstrass theorem, limit superior and limit inferior, series and convergence, Cauchy's convergence criterion, test of convergence of series with nonnegative terms, absolute and conditional convergence, alternating series, Leibniz test.

Limits of functions and sequential criterion, continuity, continuous functions on closed intervals, intermediate value theorem, uniform continuity, differentiability, Rolle's theorem and mean value theorems, Taylor's theorem, Taylor's series, Power series, radius of convergence.

Riemann Integral, the fundamental theorem of integral calculus, mean value theorems of integral calculus and applications, improper integrals and their convergence, comparison tests, absolute and conditional convergence, Abel's and Dirichlet's tests, beta and gamma functions.

6. Modern Algebra

Binary operations, groups, subgroups, normal subgroups, Lagrange's theorem, normal subgroups, quotient groups, homomorphism and isomorphism, isomorphism theorems, Cayley's Theorem, inner automorphisms, automorphisms groups, conjugacy relation, normaliser, centre of a group, class equation and Cauchy's theorem, Sylow's theorems and applications.

Rings, Integral domains, fields, subrings, characteristic of a ring, idempotent and nilpotent elements in a ring, principle, prime and maximal ideals, simple rings, definition and examples of vector space and its subspaces.

The above listing contains the topics, which are covered in BSc/BA courses in mathematics of majority of the universities. India being a vast country with many universities and institutes having BSc/BA (Honours or Major) programmes in mathematics, there are a lot of variations in the course curriculum for the programmes. The courses under the above heading in many universities go much wider and deeper.

Apart from the above courses, the programmes usually contain several other courses, some of them as elective/optional. A list of some of these courses would include:

i. Analytic Geometry of two and three dimensions
ii. Complex Analysis
iii. Mechanics – Statics, Dynamics, Hydrostatics
iv. Linear Programming, Optimisation Theory
v. Numerical Analysis
vi. Probability and Statistics
vii. Computer Programming
viii. Discrete Mathematics – Combinatorics, Graph Theory
ix. Number Theory
x. Mathematical Finance

A typical course curriculum for the BA/ BSc (General) and BCom programmes may include courses from:

1. Classical Algebra
2. Calculus
3. Analytical Geometry of two and three dimensions
4. Differential Equations
5. Modern Algebra
6. Numerical Methods
7. Linear Programming
8. Probability & Statistics
9. Computer Science & Programming
10. Discrete Mathematics
11. Mechanics

Normally, these programmes have five to nine courses in mathematics covering many of the topics listed above.

Several institutions, for example, Indian Institute of Science, Bangalore (IISc)[30], University of Hyderabad[31], IISERs, NISERs, IIT Bombay, and CMI, have four-year BS/ five-year Integrated MSc programmes which offer courses on Topology, Algebraic Topology, Manifolds, Functional Analysis, Galois Theory, Harmonic Analysis, Lie Groups, Fourier Analysis, Homological Algebra and Commutative Algebra at higher stages of their programmes, apart from basic courses from the above lists.

The University Grants Commission (UGC)[32] is the regulatory body of the Government of India that looks after the university education system of India. The UGC is invested with maintaining the quality of education imparted in Universities. To this end, it suggests uniform curricula for different programmes run by the universities. It carries out exercises from time to time for renewing and updating its model curricula. While the wisdom of having a uniform curriculum for all universities and institutes with various levels of resource abilities and requirements is debatable, the efforts of UGC work as an impetus for the universities to reflect on and review their respective curricula. The UGC, in an effort to improve standards decided on creating a model curriculum for the mathematics taught at the undergraduate level. The UGC recommended courses for BA/ BSc (Honours) in its Model Curriculum[33] of 2001 are as follows:

1. Algebra and Trigonometry: includes matrices, systems of linear equations, theory of equations; introductions to groups and rings; De Moivre's Theorem and its applications in trigonometry, etc.
2. Calculus: includes differential & integral calculus, ordinary differential equations of first and second order.
3. Vector Analysis and Geometry: includes Theorems of Gauss, Green and Stoke, and analytic geometry (conics and second degree equations, plane, sphere, cone, cylinder, conicoids, etc.).
4. Advanced Calculus: includes convergence of real sequence and series, continuity and uniform continuity, differentiability, mean value theorems, Taylor's Theorem; continuity of functions of two variables, partial derivatives, extrema of functions, etc.
5. Differential Equations: includes ordinary and partial differential equations, calculus of variations, variational problems with moving boundaries, etc.
6. Mechanics: includes statics and dynamics.
7. Analysis: includes Real Analysis (Riemann integral, improper integral; series of arbitrary terms, double series; partial derivatives, Schwarz and Young Theorems, implicit function theorem; Fourier series) and Complex Analysis (continuity, differentiability, analytic functions, conformal mappings, etc.) and Metric spaces.
8. Abstract Algebra: includes groups, rings, vector spaces (covering also inner products, orthogonalisation, etc.) and modules.

[30] See http://www.iisc.ernet.in/ug/about.htm.

[31] See http://www.uohyd.ac.in/.

[32] See http://www.ugc.ac.in/ for more on the UGC.

[33] See http://www.ugc.ac.in/policy/maths.pdf for details.

9. Programming in C and Numerical Analysis: includes numerical integration and approximation; Monte Carlo Methods.
10. Probability Theory and Optimisation.

Each of the above was suggested as the topics for two courses to be spread over two semesters. Apart from these compulsory courses, several optional courses are suggested out of which a student is to select two. These are:

i. Principles of Computer Science
ii. Differential Geometry
iii. Discrete mathematics
iv. Mechanics (Dynamics of rigid bodies, hydrostatics)
v. Mathematical Modeling
vi. Application of Mathematics in Finance and Insurance
vii. Special Theory of Relativity
viii. Elementary Number Theory
ix. Combinatorial Number Theory
x. Computational Mathematics Laboratory

For BA/ BSc (General) programmes, UGC recommends compulsory courses as in (1-8) of the above list and the rest as optional courses along with courses chosen from (i-x) above.

The model curriculum document of the UGC also gives a comprehensive set of references for each of the courses. By and large, these are a mix of modern books some catering exclusively to an Indian audience while some are books that are used world-wide. However, an intriguing fact that one notices when one looks at the recommended books in detail is that some of the books followed in the 1960s during Raghunathan's undergraduate days like books on trigonometry, statics and dynamics by S. L. Loney and Shanti Narayan's Mathematical Analysis are still in use. So after more than 50 years, there is a lot of change in many quarters but none in some. The scenario clearly indicates that some core areas such as Calculus, Basic Algebra and Geometry have necessarily been part of the curriculum, irrespective of the time frame, and some old classics continue to prove their utility as quality texts. On the other hand, some new titles such as Contemporary Abstract Algebra by Gallian, Abstract Algebra by Dummit and Foote, Linear Algebra: a Geometric Approach and Topology of Metric Spaces both by S. Kumaresan, Real analysis by Carothers, etc., have proved their merit as textbooks for the respective courses.

The main reform in syllabi seems to have taken place in the 70s all across India. Since then however, the undergraduate curriculum in mathematics in most of the universities has not undergone any paradigm shift, either in approach or in contents. Many of the courses cover topics in both width and depth, but they are too compartmentalised. The relevance of the courses to other branches of science, technology or social sciences is not emphasised or demonstrated. Not only do they seem to be isolated bundles of knowledge far away from other areas, but they also lack in interactions among themselves. Therefore, the curriculum does not equip the student with applicability of mathematics in the scenario of modern scientific and technical developments.

Even the UGC recommendations seem to have failed in giving leadership in adapting to changing requirements in mathematics education at the undergraduate level in terms of applicability of mathematics on the one hand and the role of technology in mathematics education on the other. It is conspicuous that UGC's recommendation places the more applicable courses, namely, Programming in C, Numerical Analysis, Probability Theory and Optimisation as optional courses for BSc (General), whereas it retains all classical courses as compulsory.

Moreover, the use of Information Technology (IT) in mathematics seems to have bypassed the vast majority of such programmes across Universities in India. Even teaching the use of spreadsheet programmes, Computer Algebra Systems (CAS), etc., to aid in understanding and visualising mathematics, developing good programming skills to help model and analyse mathematics is certainly not part of the mainstream of undergraduate mathematics curriculum in India.

There are a few universities that have tried to buck this trend. For example, University of Delhi in its undergraduate mathematics curriculum has laid emphasis on applications of mathematics through mathematical modeling and use of tools like Matlab, Mathematica and Maple for studying different courses and also through the use of newer books that try to integrate the pure and applicable side of mathematics. Some of the newer Universities like Ambedkar University, Delhi (AUD)[34] and Shiv Nadar University (SNU)[35] are trying to create undergraduate mathematics curricula and use teaching methods that integrate applicability, help foster team work and give all students an opportunity to appreciate both 'pure mathematics' and 'applications'. Further, communication and presentation skills, computational skills, as well as linkages to other disciplines are also explored. There may be many more such initiatives that are not listed here. It would be interesting in the years to come to see if they succeed in filling the lacuna in the current programmes.

[34] See www.aud.ac.in.

[35] See snu.edu.in.

A brief analysis of current mathematics pedagogy at the undergraduate level

Probably, among all the teaching and learning aspects in India, the scenario of mathematics education at the undergraduate level is most alarming. In most of the colleges, 'teaching' means merely demonstrating the content, usually by 'stating and proving a theorem', and 'learning' is the same as the ability to reproduce such proofs. There are minimal interactions in the classroom with students taking a passive role. Students are seldom trained to work things out themselves. The objectives of teaching become merely to prepare the student for assessments, where she is expected to reproduce after rehearsing whatever the teacher has demonstrated. Classroom teaching and student's learning are reduced merely to preparation of the student for performance in such assessments. There is a limited scope of development of the mathematical abilities of the student. This in turn affects the very objective of preparing the student to meet the challenges in her future pursuit of higher studies or participation in industry and society.

Such teaching soon becomes counter-productive. Through the programme, the student is neither prepared with mathematical maturity for the pursuit of higher studies, nor trained with mathematical know-how for being capable of applying mathematics in other fields. On the contrary, intuition and original creativity of the student is lost in the mechanical teaching-learning process. The performance of students at national level tests like JAM[36], TIFR entrance, PG entrance tests at various universities, or for national level Postgraduate scholarships clearly reveals the fact that the teaching-learning processes at undergraduate level requires much more attention.

Assessment in the undergraduate programmes of universities is largely limited to summative assessments through semester end or annual examinations conducted centrally by the universities. Some of the universities allow limited formative assessment in the form of internal assessment at the college level. However, most research institutions have both formative and summative assessments for their programmes.

Usually, the university annual and semester end examinations assess mainly students' ability to reproduce the material provided in the text books, almost in a 'state and prove' fashion. In many of the universities, the process becomes further unchallenging because of the fact that the same set of questions are repeated in the examinations almost routinely over the years. The result is devastating. Often even a student scoring very high marks is unable to answer simple questions like giving definitions, examples or counter-examples. No easy solution exists but no solution will exist without well-trained faculty equipped to use teaching and assessment methods that will help meet the goals that every undergraduate degree should have.

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[36] See http://gate.iitkgp.ac.in/ for more details regarding JAM.

Given the scenario described above it is certainly a foregone conclusion that well trained faculty just does not exist in the proportions required. Though this is true for most parts of the country, the situation in some regions is much more alarming. For example, the entire North-East region is facing acute shortage of qualified teachers at all levels.

So how do we begin to create a pool of well-qualified, motivated faculty? We should first investigate what pre-service qualifications are required in order to be able to teach at the undergraduate level. In-service training, teaching methods that are broadly prevalent also need to be considered. The means and methods prescribed by UGC to improve quality of teachers and teaching at the undergraduate level are also analysed here.

Pre-service qualifications and training for teachers at the Undergraduate Level

The University Grants Commission (UGC) has the role of maintaining the quality of university education and sets the qualifications for faculty to be able to teach mathematics in the universities (public or private). The minimum eligibility for teaching mathematics at the undergraduate level and for a job as an Assistant Professor in post-graduate University departments, as prescribed by UGC, is a Masters in Mathematics with at least 55% marks and a National/State Eligibility Test (NET[37]/SET) certificate in Mathematics or allied subjects.

It is interesting to note that the NET was recommended as a solution in 1983 by a UGC committee in order to battle dropping standards amongst the teaching fraternity in higher education. The Committee under the chairpersonship of Professor RC Mehrotra was instituted by the UGC to review pay scales of teachers at Universities and Colleges and recommended the following[38] for the post of lecturer (Assistant Professor):

> i) Qualifying at the National test conducted for the purpose by UGC or any other agency approved by UGC.
> ii) Master's degree with at least 55 % marks or its equivalent grade.
> The qualifications should not be relaxed even for candidates possessing M.Phil/Ph.D. at the time of recruitment.

UGC and allied agencies have been conducting NET since 1989, but PhDs are now exempted from NET/ SET. At the same time the UGC has also stipulated rules regarding PhD degrees in order to improve the quality of PhDs.

The Committee in 1983 in fact went on to say that "the stipulation of M.Phil/Ph.D as an essential qualification for Lecturers had neither been followed faithfully nor did it necessarily contribute to the raising of teaching and research standards. In fact, it was of the view that, if at all, it had led to the dilution of research standards on account of the

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[37] See http://www.ugc.ac.in/inside/net.html for more details regarding NET.

[38] Taken from http://www.ugc.ac.in/inside/net.html _review%20_mungekar.pdf.

rush to get a research degree in the shortest possible time".

The NET examination does work as a filtration system, however it has not succeeded in improving the quality of the entrants into the teaching profession at Universities. The quality of MPhils and PhDs is also something to worry about. The irony now is that there are coaching classes catering to candidates who wish to clear NET.

Most departments catering to post-graduate degrees would have faculty having a higher qualification than Masters and NET. Another point to be kept in mind is that the faculty involved in teaching at the undergraduate level in mathematics is usually separate from those involved in teaching mathematics at the post-graduate levels. The result is that the pre-service qualification of faculty involved in teaching mathematics at the undergraduate level is usually just Masters and NET or at most an MPhil degree in mathematics or a related field. Most faculty members involved in teaching at the undergraduate level have very limited exposure to research in mathematics. While it is possible for faculty members to improve their qualifications while in service, the majority of faculty members do not do so as they do not see any incentive in doing so. The only in-service requirement is for faculty to do two or three 3-week refresher courses. This is usually a mandatory requirement for faculty of public universities for purposes of career advancement or promotions. The quality of such courses is very variable, most serve to just provide a certificate to aid promotional aspects and do not in anyway 'refresh' the faculty.

Unless there are in-service incentives and disincentives in place that require faculty members to improve their qualifications one does not expect any radical change in the knowledge reposited with faculty or their own experience in terms of research. It is also unlikely that the assessment patterns or teaching methods will change without incentives-disincentives built in at the policy level.

Apart from improving qualifications which is crucial, faculty also need to be aware of how Information Technology (IT) can be used effectively to motivate and attract students. Being able to visualise and simulate mathematics using computers and software can add substantially to the learning experience provided that it is done with care. Programming skills would also enhance the ability of the student to analyse mathematical problems, model problems, and search for patterns. All these would then help to formulate solutions and could even lead to new research. This way of integrating the power of IT with mathematics would certainly require in-service training of existing faculty. There are also a large number of textbooks written for undergraduate students that look at teaching and learning of mathematics in a new way. These books are easy to read, link mathematical results with applications in the real world and also have a large number of problems and projects that aid in understanding the deeper concepts. They also invariably have net-based resources to help student visualise where possible and provide students with situations that aid hands-on investigation of the concepts involved through IT.

Assessment plays multiple roles: guiding the teacher on the manner in which students have learnt what has been taught, guiding the student on the extent to which she is making progress and guiding a future teacher and/ or employer on what knowledge and skills have been acquired. In the Indian context, the last role, with an emphasis on marks and grades, tends to become the primary focus for students as subsequent admissions or employment seem to be directly dependent on these. Since students place such premium on doing well in assessments, it should be turned into a vehicle that actually covers all aspects. It should be possible to create assessment scenarios, which make sure that grades and marks are linked to actual learning and ability to apply the concepts learned. Specifically, assessments should be used to guide a multi-tier/multi-stream approach to undergraduate education without attaching a stigma of failure to those in the slower streams (or a misplaced sense of achievement among students placed on faster tracks!).

Strengthening tertiary mathematics

At the tertiary level, there are several innovative programmes that impart training in mathematics. These help to strengthen the formal system learning mathematics at colleges and universities. Many of these activities are possible due to the support, financial and otherwise received from the National Board for Higher Mathematics (NBHM)[39].

The NBHM was set up by the Government of India under the Department of Atomic Energy (DAE), in the year 1983, to foster the development of higher mathematics in the country, to formulate policies for the development of mathematics, help in the establishment and development of mathematical centres and give financial assistance to research projects and to doctoral and postdoctoral scholars. NBHM functions essentially autonomously framing its own budget taking into account the funds made available by DAE.

The role of NBHM in improving mathematics at the tertiary level and other important activities that have helped students and faculty are considered here. These include, the efforts made under Mathematics Olympiads training, the Madhava Mathematics Competition, Mathematics Training and Talent Search (MTTS) Programme and Advanced Training in Mathematics (ATM) Schools. All of these activities are funded by the NBHM.

The aims and objectives of these activities, their salient features, the numbers covered through these activities, the impact these activities have had over a period of time are presented in this section.

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[39] See http://www.nbhm.dae.gov.in/.

Mathematics Olympiads

Mathematical competitions have been held in India for a reasonably long time. Various organisations in different regions have been conducting competitions for school children on their own initiative. After the constitution of the NBHM, all these competitions were given a national coordination and the Indian National Mathematical Olympiad (INMO)[40] was started in 1986. Simultaneously, India also started preparation to send teams to the International Mathematical Olympiad (IMO)[41]. It sent its first team in 1989 to Germany and since then India has been participating consistently in IMO.

The foremost aim of the olympiad is to find mathematically gifted students among the huge population of India. Many talented students in our country are not even aware of their interest in mathematics. Mathematical Olympiad is aimed at spotting these talented children and aims to nurture their talent so that they can pursue a career in mathematics. In the backdrop of the peculiar socio-economic situation in India where professional courses are held in high esteem, olympiads help children find their real interest and some counseling helps these children to pursue mathematics. However, the Mathematical Olympiad also helps to find really gifted children who can compete with other children of the same age group from different parts of the world through IMO. The selection of a team to represent India in IMO is also the objective, but the primary objective is to nurture talent.

Since its participation in IMO in 1989, Indian contestants have been doing reasonably well in the IMO. So far 136 students have contested in IMOs during the last 23 years. Among them, nine students have won gold medals; fifty-five students have won silver; and fifty-one students have won bronze medals.

The main impact of the Mathematical Olympiad in India is the awareness it has brought. Students interested in mathematics find that a career in mathematics is not the last option. They learn that there are good institutions in India where they can study higher mathematics leading to very good career options in academic and research institutions, research establishments and industries. A large number of students who write INMO take up mathematics in their undergraduate studies. NBHM provides financial assistance to those who would like to pursue mathematics in India. Several students have also taken up allied areas like computer science and are pursuing theoretical problems in computer applications. Quite a few of the medalists have finished their doctorate and are now in good academic positions. Mathematical Olympiads have also helped in raising the standard at the school level. There are several teacher-training programmes to equip teachers in problem solving. These help teachers and in turn their students. With more awareness among the students, teachers too have to do their 'homework properly' and

[40] See http://en.wikipedia.org/wiki/Indian_National_Mathematical_Olympiad.
[41] See http://www.imo-official.org/.

prepare better. It is definitely the case that such activities increase the mathematical reasoning of the children.

Madhava Mathematics Competition

The success of Mathematics Olympiads provided a strong motivation for having a similar competition at the undergraduate level. The purpose of having a Mathematics Competition at UG level is multifold:

1. The competition would cultivate a culture of 'problem-solving' among the UG students.
2. The competition would provide a motivation for the interested students and teachers to go beyond syllabi and work on more challenging problems.
3. The competition would help in identifying the better undergraduate students and in turn would allow us to design the mechanism of nurturing them in a more systematic way.
4. The competition would generate interest and enthusiasm among the students and could help in attracting good students to Mathematics.
5. The participating institutions would be linked through the competition and a possibility of a meaningful interaction between them would evolve.

The competition is still in its early stages but it is rapidly growing as a national level competition for math undergraduates in India. The Centre for Postgraduate Studies in Mathematics, S. P. College[42], Pune organises the competition jointly with the Homi Bhabha Centre for Science Education, Mumbai.

A distinctive feature of the competition is organisation of a nurture camp for the prize winners. The camp is organised in the month of May at Bhaskaracharya Pratishthana[43], Pune. Though at present the competition is conducted in some parts of the country, it has been planned to hold it countrywide in near future.

Mathematics Training and Talent Search (MTTS) programme[44]

Mathematics Training and Talent Search Programme (MTTS) is a national level four weeks intensive summer training programme in mathematics. It has been being organised in every summer for the last 19 years (since 1993) in India. This programme was conceived and has been directed since its inception by Professor S. Kumaresan[45], now at

[42] See http://www.spcollegepune.ac.in/newsite/.
[43] See http://www.bprim.org/.
[44] See http://www.mtts.org.in/.
[45] See http://mathstat.uohyd.ernet.in/people/kumaresan/More-about-Kumaresan for more about Professor Kumaresan.

the University of Hyderabad. It is one of the most significant and successful mathematics training programmes and has made an impressive impact on mathematical scene in India over the years, especially at undergraduate and post-graduate levels. Each year about 180-190 talented students selected from all over India, undergo this training programme at three different levels at several centres across the country.

The programme aims to teach mathematics in an interactive way and to develop independent thinking in mathematics among the participants. To promote active learning, the teachers usually ask questions and try to develop the theory based on the answers and typical examples. At every level the participants are encouraged to explore, guess and formulate definitions and results. Moreover, the programme provides a platform for the talented students so that they can interact with their peers and experts in the field. This serves two purposes: i) the participants come to know where they stand academically and what they have to do to bring out their full potential and ii) they establish a rapport with other participants and teachers which help them shape their career in mathematics.

The MTTS Programme has made some significant impact in the scenario of mathematics education in India, especially at the Post-Graduate level. Out of a total of more than 2700 participants, about 300 students pursued higher studies. Many of the participants of the programme, who at present are engaged in teaching and research in the premier institutions of the country, acknowledge that their attitude towards mathematics was transformed by the programme. Apart from its contribution in research, the programme has also produced some good teachers for school and college education, though the number is small relative to the requirement for the country.

The success of MTTS has resulted in the organising of a similar training programme for teachers titled Pedagogical Training for Mathematics Teachers (PTMT) from the year 2012.

Advanced Training in Mathematics (ATM)[46]

There are a large number of doctoral students, postdoctoral fellows and faculty members in universities and various institutions of higher learning for mathematics in India. It was felt that there is a need for programmes that will help such scholars improve their knowledge base, and would also strengthen and broaden areas of research that this body of scholars can engage in. To this end the NBHM launched ATM Schools in the inter-related areas of algebra, analysis, partial differential equations, discrete mathematics, geometry, number theory and topology. The aim of this integrated training programme is to provide basic and advanced knowledge in these areas and emphasise their interconnections via a series of instructional schools.

A highlight of the instructional school is a series of lectures for a week by an eminent

[46] See http://www.atmschools.org/.

mathematician under the title *Unity of Mathematics Lectures*. These instructional schools are also held at different levels for different target audiences. Of particular importance are instructional schools devoted to imparting training to college and university teachers. So far over 750 college teachers and about 3000 PhD and MSc students have been beneficiaries of the training under ATM.

Conclusion

There are about 400 Universities and 18000 colleges (including Engineering and Polytechnics) in India where the teaching and learning of mathematics takes place. Out of around 2,000,000 students enrolled for undergraduate courses, about 400,000 students enroll for post-graduate courses. The estimated number of students pursuing post-graduation in mathematics is around 25,000. The number of students pursuing MPhil or PhD in mathematics is in the range of 800-1000 and there are about 30,000 teachers working at the undergraduate or post-graduate levels.

While the numbers involved are large, these make up only a tiny fraction of the populace. It is clear that for a country that is making great strides in many fields, the lack of an educated work-force will prove to be a huge speed-breaker.

An analysis of the past and the present shows us several things that India can be rightly proud of but at the same time cautions against any complacence. It is clear that the mathematics in the undergraduate mathematics curriculum has broadly kept up with international standards. Even the model curriculum of the UGC does have plenty of courses from what would be termed as 'modern mathematics'. However, the teaching-learning process at the undergraduate level is not even meeting what should be its minimum goals.

It is true that the average undergraduate mathematics student now has access to far more books, information and access to computers and computer networks. They also have a reasonably good curriculum to study from. The average qualification of a faculty member teaching at the undergraduate level is better than what was the case several decades ago. More women students are doing mathematics than ever before. In urban centres, half the mathematics class is usually women and this ratio improves further in taught post-graduate courses in mathematics. These are all positives that we can be justifiably happy about.

However given the decade we are currently in and the growing needs of our society and the needs of the discipline itself, unless we take strong ameliorative steps the rate at which we are improving is just not going to be enough. If we take a closer look we actually see the many gaps and lacunae that require immediate healing. There is a requirement to both work out long-term strategies and at the same time to also have good achievable short-term goals. Given the diversity and size of India there have to be a multiplicity of approaches rather than a single quick fix.

To sum up, the curriculum in most of the high weightage undergraduate mathematics programmes seem to be focused on fast-tracking young men and women to be research mathematicians. On average however much less than a fourth of undergraduate mathematics students actually decide to pursue an academic career in mathematics. Further the pedagogy and assessment patterns followed actually do not do much to foster or enhance the ability to think originally or to critically analyse and solve unseen questions. Thus on average the undergraduate programmes in mathematics fail in at least two important ways: one they are not really equipping and training the minority that plan to take up a career in mathematics in the manner they should; two, the majority are neither gaining any understanding of the role of mathematics in society nor are they learning the skills required by all in terms of communication, presentation, or the use of modern computer technology.

A solution to this is certainly possible. On the curricular front we need to create a syllabi that through its content, recommended books and resource material would make learning mathematics meaningful in more ways than one. Improved qualifications, focused in-service training for faculty particularly in terms of familiarity with programming and use of mathematical software, improved infra-structure, and well conceived schemes of both incentives and disincentives can create a pool of faculty members who are equipped to use innovative teaching methods to impart the curriculum. Further the schemes for strengthening tertiary mathematics need to be scaled up and need to spread to smaller towns and rural districts. Special attention also needs to be given to attracting more students and also more women students to research.

The existing hierarchies in education have created compartmentalised discrete structures that mitigate against continuous flow of information and ideas between different levels of mathematics. There also seems to be almost no data capturing the state of undergraduate mathematics education. There is no significant research undertaken about undergraduate or tertiary mathematics education. By and large the community of mathematicians and mathematics educators in India seem to inhabit separate worlds. This too needs to change. Improvement just at the undergraduate or tertiary level is not enough. The entire community needs to focus on improving mathematics education at all levels. Seminars, conferences and research can go a long way in creating the necessary paths that lead to a better understanding of the problems. It will also help in framing policy that will hopefully pave the way and provide the right setting for the solutions to take root.

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