10. Mathematics education research in India: Issues and challenges

Rakhi Banerjee
Bharat Ratna Dr. B . R. Ambedkar University, Delhi
rakhi@aud.ac.in

Background

Mathematics education research, the world over, has tried to study a variety of issues. These have included understanding construction of mathematical knowledge among students, teachers, various groups and communities; understanding how they acquire this mathematical knowledge and use it to think and organize their experiences or organize their teaching; understanding ways in which social, political, economic and ideological factors influence the curriculum, content, teaching and learning of mathematics and thus access to mathematics for all children/ students. Many studies have also been conducted to understand the relation between mathematics (as a discipline, its epistemology, history) and mathematics education. In this context, we see specific studies having been conducted with regard to issues related to the content of mathematics at different levels of education, processes involved in learning mathematics, and how different aspects of the classroom culture/process contribute to the acquisition of mathematical knowledge (Sierpinska and Kilpatrick, 1998). Mathematics education research has evolved over decades of research, starting from Thorndike and the many critiques of his theory which focused on meaningful and purposeful mathematics to complex research designs and multiple disciplinary frameworks in order to explore the issues impinging on teaching and learning of mathematics. Many theories have been adapted and modified from other disciplines and theories of teaching and learning have been formulated in mathematics education. However, this development has not reached or is not shared uniformly across the globe.

Mathematics education and research in mathematics education

The place of mathematics as a subject in the school curriculum in India has always been valued and the need for improvement in its content and pedagogy has been emphasized by various commissions on education constituted by the Government of India. The

progressive tone that is seen elsewhere in the world is also visible in this country. Time and again a need has been expressed in these documents of taking mathematics beyond mechanical computations and focus on understanding basic principles. Kapur (1997) in the Fifth Survey of Education Research (1988-92) had pointed out that "the main object of mathematics education research is to be of help in improvement of classroom learning and teaching. It is therefore natural that a large number of studies should be concerned with different aspects of this problem". He went on to suggest various dimensions of this research, including study of effective teachers, instruction based on the use of computer aided and other technology, error analysis, styles of learning among different groups of students, remedial teaching methods, study of attitudes, socio-economic and other personal factors influencing learning. More than two decades later, many of these issues, in their true sense (in the way we understand the domain and purposes of mathematics education and research in mathematics education), are yet to be researched.

There have been many initiatives and interventions in the area of mathematics teaching and learning but they have not been documented and analyzed; nor do we have enough illustration of the understanding, thinking and reasoning of students participating in these initiatives (See Chapter 9, this volume). What we know about their success (or not so successful attempts) is more anecdotal than based on systematic investigation. However, their contribution to the mathematics education scene in the country is immense. They explicated a philosophy of teaching and learning based on an understanding of the child and his/ her capacities to learn and think. These earlier attempts and deliberations made possible new ways of thinking about teaching and learning of mathematics that are seen today and have become part of the National Curriculum Framework (NCF-2005) (National Council for Educational Research and Training [NCERT], 2005) and the new textbooks developed after this.

Typically, the departments of education in various state universities, some private universities and the NCERT along with the five Regional Institutes of Education (RIE) set up by the NCERT (to cater to specific regions and states of the country) are involved in carrying out research in the area of education, including subject specific research, like mathematics. In general, the mathematics departments (or any other discipline based department) of these universities are not involved in research in the area of mathematics *education* at any level. The education departments have traditionally taken on the task of in-service and pre-service teacher education and in this endeavour are isolated from other departments of the university, both structurally and intellectually. Each department may have one or just a few faculty members with mathematics as their area of specialization. These departments offer undergraduate (mandatory pre-service teacher education programme for teaching at the secondary school level), postgraduate degrees in education and pre-doctoral and doctoral level programmes, with an emphasis on educational studies. Largely, students graduating from the postgraduate programme take up a teaching or

an administrative position in colleges, universities or schools. (There are many more avenues open today, with more and more state as well as commercial companies putting in large money in education.) Very few of these postgraduate students get trained in the departments for research in the area of education, and even fewer in mathematics education. The strict entry qualifications as well as mandates of the departments do not allow easy movement of students from departments of education to departments of mathematics and vice-versa at postgraduate and research levels.

Given this scenario, it is not difficult to understand that research in the area of mathematics education has not been a high point in the country. A large amount of research that has been carried out has focused on very traditional psychometric models. There is some evolution which research in mathematics education has gone through in the last decade and there is an attempt to study and explore some issues in the teaching and learning of mathematics. A synopsis of studies of both kinds will be presented in the next section.

Traditional research in mathematics education in India

A review of some of the educational abstracts and surveys of education (Buch, Joshi and National Council of Educational Research and Training, 1991; Kapur, 1997; NCERT, 1999) reveal the following trend in research in mathematics education.

A large number of research studies in India have focused on designing diagnostic tests in order to identify learning difficulties in some content area or the other and standardizing them. These studies in general are followed by remedial teaching; there are other studies which only design a remedial teaching approach after administering a suitable standardized test. However, the thesis, reports or abstracts available describing these attempts do not give very specific insights into the test items, the approach adopted for remedial teaching or illustration of students' understanding of any concept or idea. Largely, the tests led to the identification of difficulties students face in learning various ideas in mathematics, and the remedial teaching produced significant difference in students' performances.

- Exploratory studies tried to understand the factors responsible for poor achievement or failure in mathematics. The design of these studies generally included the use of standardized scales/ tests followed by statistical analysis to find possible correlates/ factors. The following factors were found:
    - Lack of pre-requisites
    - Difficulties with understanding the language
    - Difficulties due to certain kinds of teaching
    - Mathematization of verbal problems and the interpretation of solutions
    - Socio-economic factors

    - Intelligence, attitude, study habits, reasoning power, spatial visualization
- Studies also tried to list the kinds of errors which students commit while working on problems in certain areas.
- Studies explored reasons for mathematics anxiety and fear among students, language issues in mathematics teaching and learning, nature of mathematics and its pedagogical implication.
- Comparison of different methods of teaching, development of activities and kits for activity centred learning, functionality/ need of some policy (like the prescription of minimum levels of learning for each grade level) or some feature like the mathematics laboratory have also been studied.

This trend of research has been highly governed by psychometric designs and models of data analysis and has dominated the Indian mathematics education research scene. Some of the studies in this tradition are no doubt significant, but they are much limited by their methodologies and outlook. Thus, we have not been able to develop a deeper understanding of the issues in a way that is useful for designing policies on teaching and learning of mathematics or for designing the curriculum. Statistical significances achieved with standardized tests and scales are not always sufficient to grasp the complexity of the matter of study. We need to develop multiple theoretical frameworks and sufficient empirical basis to study and intervene in the problems of mathematics education. For example, we need to understand how say, socio-economic factors or language or reasoning influence teaching and learning of mathematics. What kinds of teacher actions, curriculum materials or classroom environment are responsible for discrimination against certain sections of society and leads to students' losing interest in the subject or dropping out of school? What is needed is increasing the depth, quality and variety of the studies conducted rather than large sample, survey kind of data on which certain statistical techniques are employed to derive conclusions.

New trends in mathematics education research in India

Over the years, however, we do see some change in this paradigm of research, especially in some studies after the year 2000. These studies/approaches are more analytical in nature, make efforts to understand the teaching or the learning process through designs of study other than experimental methods. Incidentally, many conferences focusing on science, technology and mathematics education have been held around this time in the country (e.g. CASTME-UNESCO-HBCSE International conference, February 2001, epiSTEME conferences held every two years by the Homi Bhabha Centre for Science Education, Mumbai), some of which have had an international character and gave an opportunity for researchers and practitioners to share ideas with a wider audience. Two

conferences (First and Second conference on mathematical education in South Asia) held in the year 1956 and 1960 at the Tata Institute of Fundamental Research, Mumbai were precursors to this modern trend of research. These early conferences were attended by a large number of eminent mathematicians from across the world, including Hans Freudenthal. They were geared towards tertiary level and research in mathematics and the discussions revolved around the teaching of various branches of mathematics in the light of modern developments and hoped to provide an impetus to teachers of mathematics in South Asia. In the first conference, Freudenthal spoke about Realistic Mathematics Education. Having been hosted in the country's premier research institute in fundamental sciences, the deliberations and ideas generated in these conferences did not reach the wider society and universities in the country. It is only now that we are trying to catch up with the developments in mathematics education research worldwide. Some efforts have been made by individuals in the recent years to submit and present research papers in international conferences like the Psychology of Mathematics Education and the International Congress for Mathematical Education and submit research based articles in journals of repute. I will try to illustrate the nature and details of these studies in the following paragraphs.

Teaching and learning of mathematics in the elementary grades

In a large country as ours and the few research studies that have been conducted so far, elementary mathematics has been the most researched area. Groups or individuals have engaged with the crucial areas of elementary mathematics, both in designing intervention studies as well as conducting exploratory studies in order to understand different aspects of teaching and learning of mathematics.

Teaching interventions

Intervention studies have been conducted at the primary and the middle school level in order to increase students' understanding of particular areas of mathematics like number concepts and operations, geometry, fractions, algebra. Different groups of individuals have made the effort of designing alternative routes of learning these areas and simultaneously placed them in the context of trends in international research. The highlight of these studies is the detailed discussion of students' understanding of the concepts within the teaching-learning situations. Menon (2004, 2007, 2009) has been trying to explore possible alternatives to a variety of topics in primary mathematics curriculum like angles in geometry, place value and understanding of numbers and word problems. This series of studies is highly influenced by the Realistic Mathematics Education approach developed by the Freudenthal Institute (van den Heuvel-Panhuizen, 2001) with which Menon combines a Vygotskian perspective (Menon, 2009). In her

efforts to develop an alternative geometry trajectory, she reported on grade 1 and 2 children's understanding of angles, which was found to be very much within their zone of proximal development. The author thus made a case for introducing such ideas much earlier in the formal curriculum than is done currently. In another study, she looked at the place and ways in which word problems are introduced in textbooks and argued for alternative approaches. She highlighted the use of word problems for mathematization of reality so that situation specific models can be developed as tools for mathematical reasoning. She has also argued for of moving away from an analysis on the place value understanding of numbers and developing number sense using the empty number line.

Fraction has been considered to be a difficult concept and arguments have been put for dropping the topic from the primary grades; its utility being very limited in daily life and largely taught for introducing rational numbers. This has been a motivation for a few groups to explore this area more conceptually, its relevance and its different interpretation so as to make a more meaningful teaching intervention possible. Subramanian and Verma (2009), Naik and Subramaniam (2008), Subramaniam and Naik (2007) give some glimpse of such studies. These studies use one or more interpretations of fraction (measure, share/ quotient, ratio, operator) and use these as models for building children's understanding in the primary school and illustrate their reasoning in various problem situations. These interpretations give meaning to the concept of fractions and the tasks are so designed that they make it relevant to children's lives. The studies show the relative ease with which children use these interpretations/ models to think about fractions and their effectiveness vis-à-vis the part-whole (a more static) interpretation of fractions. Sankaran, Sampath and Sivaswamy (2009) tried to build a computer aided tool using the part-whole interpretation for teaching fractions and based it on the cultural aspects of the learners (building a bead necklace of different colours, tiling an area). They concluded that such an approach which uses a familiar idea to teach difficult concepts is useful and helps enhance children's conceptual understanding.

Banerjee, Subramaniam and Naik (2008) describe the evolution of a teaching approach for beginning algebra and highlight the use of arithmetic in specific ways to help students make the transition. The study used concepts like equality, terms (additive components of the expression), expressions, and value of the expression in order to make the transition from arithmetic to beginning symbolic algebra. These concepts and ideas not only helped in understanding symbolic transformations in algebra but also enabled the students to reason about numerical and algebraic expressions, with respect to their value. Moreover, these ideas also allowed them a way to think about tasks which dealt with proving and generalizing. It was somewhat natural for the students to see that in order to complete these tasks they needed to make a representation (arithmetic/ and or algebraic) and that transformations on them would lead to a value or an equivalent expression, which bore relevance to the solution of the task.

Exploratory studies

A few exploratory studies on children's understanding of numbers and basic operations on them have been conducted. In one study, Khan (2004) explored three different groups of children's understanding of these, namely *paan* (betel leaf) sellers, newspaper vendors and children going to municipal schools in Delhi. The *paan* sellers, who sell a greater variety of articles at different prices than newspaper vendors, were the better of the three groups and there was not much difference in the performance of the other two groups. However, the interesting difference was in the manner the *paan* sellers and the newspaper vendors approached the problems and solved them in comparison to the school going children. These two former groups of children were able to understand the meaning and the import of the problems, had devised many alternative ways to work with numbers during their transactions but were limited by their inability to work with all kinds of numbers; the school going group was more concerned about correct answers, than methods, processes, reasoning. Another of her studies (Khan, 2008) explored children's acquisition of number concepts, with focus on their representation in a formal language. One of the important lessons to be learnt from this study is the complexity in learning this very basic concept in mathematics and how both language (in this case Hindi) and teachers' failure to appreciate the challenges involved make things difficult for children very early in their school life.

A study to increase cognitive capacities of children through what the author called "thinking mathematics" lessons has also been conducted (Chilakammari, 2001). It used the CAME model (cognitive acceleration in and through mathematics education) and the experimental study showed that children's learning was accelerated as a result of mathematics lessons based on thinking and sharing of ideas in the classroom.

Work in progress

Similar work around other concepts and areas of primary mathematics is visible now by individuals (the regional conferences and the national conference held under the auspices of National Initiative in Mathematics Education – NIME-2011-12 – give a glimpse of such work). Not all of these have taken the shape of fully fledged research but are initial attempts in the field of mathematics education and grappling with problems in the field. Many of these studies deal with exploring students' understanding of concepts in areas like numbers, fractions, geometry, algebra, measurement and probability and difficulties students face in learning them. Investigators are trying to design alternative ways/ trajectories which help students make sense of key concepts and procedures in various content areas (numbers and basic operations, arithmetic, geometry, algebra, measurement, etc.), explore classroom dynamics, relation between curriculum, textbooks, pedagogical practices and students' learning, and teachers' knowledge through students' mathematics. Attempts are being made to use particular theories (e.g. Lesh's model of multimodal

representation) in designing teaching and learning tasks. Researchers are beginning to look at a crucial aspect of mathematics education, that of assessment, and its role in giving insights into students' learning and its possible use by teachers, designing of assessment tasks and its recording and communication. Studies on mathematics among communities and in the culture are also found. Government level initiatives have led to collaboration between some Indian mathematics educators and Swedish counterparts in the area of use of technology for mathematics teaching. These do give a ray of hope that mathematics education would find a place as a discipline and as a research domain in the country. However, these are a small number of people placed in select organizations and institutions (elite and better off institutions and organizations) and a large number of research studies in the education departments of mainstream universities still follow the old paradigm of research.

Higher secondary and higher education

Studies in the higher secondary level are even fewer. Parameswaran (2007a, 2007b, 2010) has been studying factors which influence high school students' understanding of abstract definitions like graphs or concepts like limits and infinitesimal quantities when they encounter these for the first time. One of the major factors which do not allow them to fully understand these complex ideas is students' prior learning of similar but more concrete ideas. Her study with expert mathematicians revealed the cognitive tool they employ to develop deep understanding of mathematical definitions (Parameswaran, 2010).

Efforts have also been made to systematically use e-learning platforms, graphic calculators, Computer Algebra Systems, and dynamic geometry softwares in the higher secondary classrooms to make mathematics interesting and provide challenging tasks to students and make their learning of abstract mathematics meaningful. Several small classroom experiments have been conducted with high school students in the designing of interesting projects (Ghosh, 2001, 2011; Asija, 2011, Kathuria, 2011).

Higher education has largely been a neglected area as far as mathematics education is concerned. The Indira Gandhi National Open University made some efforts to collect data on a bridge programme (the Bachelors Preparatory Programme, BPP) that they had launched to allow access to the undergraduate degree programme for people who had no formal high school leaving certificate. Feedback collected over a 3-year period suggested the need for curricular revision so as to meet the needs of the students (such as simplification of content, basic mathematics with emphasis on mathematical thinking and comfort with dealing with mathematical language) (Sinclair and Varma, 2001).

Teacher education

Teacher education is another area where there is a lot of scope for research. In India, few such studies exist, although many teacher interventions have been conceptualized and implemented. Rawool (2001, 2007) has been making efforts to document and analyze the effect of simulation of teaching practice on trainee teachers to learn to teach. The programme itself and the study tried to identify elements that are critical for helping trainee teachers to develop a theoretical framework about teaching and learning, finding motivations for student to engage in learning, designing and organizing non-traditional learning environments. This is one of the few reports that give some detailed account of thinking, reasoning and questioning that was happening among the group of trainee teachers. It tried to document the difficulties which these trainee teachers were facing, with respect to language, comprehension, their own comfort with content, etc., while participating in this pre-service programme.

Summing up trends in research

It is evident from the above description, that certain kinds of research studies have been undertaken so far. Groups or individuals have largely addressed content related issues, in the elementary grades. They have focused on exploring students' understanding of content areas covered in these grades or have designed intervention studies which help overcome the difficulties students face in learning them. Thus, both the psychometric and other designs of studies share a similarity with respect to the question(s) that have been addressed. As can be expected, the more recent studies attempt to report on students' reasoning and thinking within the situation of learning and illustrate the ways in which a particular teaching-learning situation impacts their learning, highlighting both the positive aspects and the challenges. This provides a rich set of data using which one can hope to design a curriculum, which have some theoretical and empirical basis and based on an understanding of the challenges likely to be faced in their implementation.

Further, there are hardly any research studies conducted at the secondary and senior secondary levels dealing with the issue of content and students' understanding of them. The few people who are involved at this level, have either tried to explore difficulties which students face while learning a new concept/ idea or have designed computer intensive environments in order to give students an opportunity to make sense of the mathematics they are learning. A deeper analysis of student work and their overall cognitive and affective gains in the later kind of studies would be useful for making a case for creating space for such components in the curricula.

Uncovering the gaps in research

In a country of the size and diversity of India where many socio-economic-political dynamics operate, the number of research studies in mathematics education is far too few to have any significant impact on either policy or on our own understanding of children's levels and their capacities. This is evident from the complete absence of reflection of such work in the new textbooks which were written post NCF-2005.

The above is coupled with the fact that our engagement with theories is minimal and is restricted to using one or the other theory for purposes of a study, leading to more descriptive reports than analytic ones. These theories come from various disciplinary frameworks of the social sciences as well as there are theories generated within mathematics education. The separation of education from social sciences, and making education almost like a collection of skills, increases the chances of theories being mechanically used rather than engaged with critically. The object of study being very complex, research studies in the "progressive" trend would gain by a deeper engagement with theory and bringing in some rigour in the methodology used. It is possibly this reason, that these studies do not find themselves readily used for making policy or curricular decisions.

There is also a need to engage with deeper issues of teaching and learning of mathematics at all levels and not only work in the broad areas of content of mathematics, as has been the case till now. These include issue related to representations and symbols in mathematics, meaning making, language issues, reasoning, argumentation and proving, use of technology, understanding classroom cultures, teacher education, socio-political-economic questions and its impact on mathematics education, affect and mathematics teaching and learning, and assessment. It is these issues which impinge directly on the framework we choose for designing studies or conception of a curriculum. The research studies have largely been silent on these issues or at least have not addressed them directly.

Very little attention has been paid to research in teacher education, which is one of the key areas of research in mathematics education. Much of curriculum reform or any change in teaching and learning of mathematics would be difficult to actualize as we do not clearly understand what aspects of the teachers' personality and their academic achievements are responsible for being able to teach effectively mathematics that is considered important for students to learn. Documentation of efforts made for professional development of teachers is essential to envisage a new design of curriculum and teacher preparation programme as well as in-service teacher development. Research must be able to explore characteristics of "good" teachers, its relation with their prior qualification, teachers' pedagogical content knowledge, their beliefs, attitudes towards teaching, learning and mathematics. Moreover, it is through research that we would be able to get a glimpse of teachers' lives – their daily practice, the experiences that shape their practices (both academic and non-academic decisions in the classroom), their own professional development with increasing experience as a teacher.

Issues and challenges and ways ahead

Systemic issues

One of the major challenges that India faces in developing mathematics education as a research area is the lack of systemic support. The university departments and colleges of education have not been able to provide the space and support for establishing traditions of content specific and subject specific research with sound theoretical frameworks and well designed empirical studies. This is also due to the fact that education departments are isolated from departments of subject disciplines, like mathematics, which can provide inputs on the content aspect in mathematics education research. Thus, even when lot of research studies have been conducted in the area of education (using the frameworks and theories in education), few have been conducted in mathematics education. The studies conducted in the departments still follow the traditional style of research with focus on psychometric designs, studies on lists of errors in different areas, studies in the area of fear and anxiety in mathematics, nature of mathematics and its pedagogical implication. In the recent years, one can see some change in these departments as well with some titles moving beyond the ones mentioned earlier (like, making sense of the classroom environment, teacher practice). However, the issues being very complex, they often lack theoretical and methodological rigour. Also, students who undertake the masters dissertation, which has a narrow scope, do not often get enough opportunities, despite their motivation and interest, to continue their work and contribute to the field. What one needs is to make these places rich in both print and human resources and students should be exposed to career opportunities in research in the area early on.

The studies mentioned in the discussion earlier have come from a few individuals, institutions or non-government organizations. There is an acute shortage of experienced faculty/ researchers across the country, who can take up research issues in mathematics education as well as contribute effectively to teacher education programmes. There is no systemic structure to support and strengthen such work. This becomes a vicious cycle – the fact that there are few researchers in the area makes it difficult to establish departments specific to mathematics education which can conduct research studies in the area and the lack of such departments makes it impossible to produce researchers. It is largely personal interest and motivation which drives individuals to do research in the area.

Issues related to diversity and complexity of the subject matter

In the Indian context, questions of meaning, symbols, classroom environment, technology and access to quality education are very important, given the diversity that exist across the country, in terms of physical and human resources and culture and context of people.

The debate about meaningfulness of the mathematics learnt or the mathematical activity has taken several twists and turns. Not so long ago, learning mathematics was thought to be meaningful as it provided access to prestigious professional or academic career. All children went through a certain kind of mathematics for ten years[1], which prepared them successively for the next stage, keeping the end in mind (mathematics for professionals or mathematicians). In some time, it was found that most children do not succeed in this endeavour and are therefore not able to gain through their mathematics learning/ teaching, but develop anxiety, fear, lack of confidence and hopelessness. Thus, the need arose for rethinking the mathematics curriculum. The root cause of this failure among students was identified to be the meaninglessness of the mathematical activity. The first effort to infuse meaning, through large scale governmental and non-governmental initiatives, was to increase activities, games, concrete materials and word problems (signifying application of mathematics to real world) in the mathematics classroom. Although one found overall gains in attitude and confidence of children who participated in them, no systematic attempts were made to collect data of students' learning of mathematics in these situations. All the same, many felt that this mathematics too was not very useful and the applications in terms of word problems were rather contrived. Thus, another attempt to make mathematics meaningful came by embedding mathematics in real world contexts of children and thereby engaging with critical theories and pedagogies of education (seen in the new textbooks written by NCERT).

In the process of taking any of these policy decisions, we need a more nuanced understanding of what "meaning" is – what types of activities can be considered to be meaningful, what cannot be meaningful, positioning and sequencing of these activities, emphasis on different aspects of mathematics, etc. What role do concepts and symbols play in this process? How does understanding of symbols progress? What is the relation between concepts, procedures and symbols in mathematics? What kind of classroom cultures enable children to not only make sense of the mathematics that is a lived reality for them but also transcend it and move into the world of mathematics, deal with the abstractions? What is the role of communication, reasoning and argumentation in the classroom? What strategies can be evolved to take this forward and learn to communicate in the language of mathematics, for example, writing proofs? In what ways can technology help teaching in the classroom? What is the role of the teacher in such a classroom? What kind of knowledge and preparation should the teacher have? What challenges and issues arise while implementing any of the above in the classroom?

Thus, the process of arriving at a resolution for the idea of "meaning" in mathematics is a complicated one. It immediately gets intertwined with many other things, including questions of access and quality education. If relevance or context is given priority over

[1] Mathematics has been a mandatory subject for all school going children till grade ten across the country for several decades.

ideas to be learnt in a mathematics lesson, then we have to explore the extent to which an equitable access to mathematics learning can be provided, given the diversity in the country. How would this impact their later learning? What vision do we have of a mathematics learner who would exit school after grades 8, 10 or 12? Can this be different for different kinds of learners? At what grade level can we start differentiating between students? Is it possible to completely remove considerations of the long term ends or directions students may choose to take?

These are certain issues and questions which mathematics education research must illuminate. These cut across different levels of schooling and different content areas within mathematics, thus are broad and overarching. Debates and answers to these issues are important to be able to make the kind of decisions that are taken while formulating a new curriculum or other policy decisions. Research studies undertaken must be able to address one or more of these issues, directly or indirectly. It is only on the basis of theoretical and empirical research dealing with critical issues such as these that we will be able to make more informed policies and be able to better analyse our existing policies.

References

Asija, H. (2011). Developing Project and Inquiry Based Learning in Mathematics at the Secondary School. Presentation made at the National Initiative on Mathematics Education – Northern Region Conference, Delhi.

Banerjee, R., Subramaniam, K. and Naik, S. (2008). Briding Arithmetic and Algebra: Evolution of a Teaching Sequence. In O. Figueras et al. (Eds.), *International group of the psychology of mathematics education: Proceedings of the Joint Meeting of PME 32 and PME-NA XXX* (PME29), Vol 2, 121-128, Morelia, Mexico.

Buch, M. B., Joshi, J. N., National Council of Educational Research and Training. (1991). *Fourth survey of educational research (1983-88)*. New Delhi: NCERT.

Chilakammari, V. (2001). Cognitive acceleration in mathematics education. In S. C. Agarkar, V. D. Lale (Eds.) *Proceedings of the CASTME-UNESCO-HBCSE International Conference*. Goa, India: HBCSE.

Ghosh, J. B. (2001) Teaching applications of secondary school mathematics with computer algebra. In S. C. Agarkar, V. D. Lale (Eds.) *Proceedings of the CASTME-UNESCO-HBCSE International Conference*. Goa, India: HBCSE.

Ghosh, J. B. (2011). Technology in mathematics education. Presentation made at the National Initiative on Mathematics Education – Northern Region Conference, Delhi.

Kapur, J. N. (1997). Mathematics Education. In National Council of Educational Research and Training (Eds.) *Fifth survey of educational research (1988-92), Trend reports (Vol. 1)*. New Delhi: NCERT.

Kathuria, R. (2011). Curriculum and pedagogy at middle and secondary school level. Presentation made at the National Initiative on Mathematics Education – Northern Region Conference, Delhi.

Khan, F. A. (2004). Living, learning and doing mathematics: a study of working-class children in Delhi. *Contemporary education dialogue,* Vol. 1(2), pp. 199-227.

Khan, F. A. (2008). Formal number systems in the context of early schooling. *Contemporary education dialogue,* Vol. 6(1), pp. 5-24.

Menon, U. (2004) The teaching of place value – cognitive considerations. Presentation made at epiSTEME-1 conference, Goa, India.

Menon, U. (2007) Word problems and activities – designing the curriculum. In C. Natarajan and B. Choksi (Eds.) *Proceedings of epiSTEME-2 conference*. Mumbai: HBCSE.

Menon, U. (2009). The introduction of angles. In K. Subramaniam and A. Mazumdar (Eds.) *Proceedings of epiSTEME-3 conference*. Mumbai: HBCSE.

Naik, S. and Subramaniam, K. (2008). Integrating the measure and quotient interpretation of fractions. In O. Figueras et al. (Eds.), *International group of the psychology of mathematics education: Proceedings of the Joint Meeting of PME 32 and PME-NA XXX (PME29)*, Vol 4, 17-24, Morelia, Mexico.

National Council of Educational Research and Training. (1998). *Indian Educational Abstracts, Issue 5*. New Delhi: NCERT.

National Council of Educational Research and Training. (1999). *Indian Educational Abstracts, Issue 6*. New Delhi: NCERT.

National Council for Educational Research and Training. (2005). *National curriculum framework*. Delhi: NCERT publications.

Parameswaran, R. (2007a) Understanding abstract definitions: graphs, a case study. In C. Natarajan and B. Choksi (Eds.) *Proceedings of epiSTEME-2 conference*. Mumbai: HBCSE.

Parameswaran, R. (2007b) On understanding the notion of limits and infinitesimal quantities. *International journal of science and mathematics education*, Vol. 5(2), pp. 193-216.

Parameswaran, R. (2010) Expert mathematician's approach to understanding definitions, *The mathematics educator,* Vol. 20(1), pp. 43-51.

Rawool, S. (2001). Designing learning activities for developing mathematical concepts in collaboration with student teachers. In S. C. Agarkar, V. D. Lale (Eds.) *Proceedings of the CASTME-UNESCO-HBCSE International Conference*. Goa, India: HBCSE.

Rawool, S. (2007). Significant and cognitive learning of mathematics. In C. Natarajan and B. Choksi (Eds.) *Proceedings of epiSTEME-2 conference*. Mumbai: HBCSE.

Sankaran, S., Sampath, H. and Sivaswamy, J. (2009). In Kong, S.C., Ogata, H., Arn-

seth, H.C., Chan, C.K.K., Hirashima, T., Klett, F., Lee, J.H.M., Liu, C.C., Looi, C.K., Milrad, M., Mitrovic, A., Nakabayashi, K., Wong, S.L., Yang, S.J.H. (Eds.) *Proceedings of the 17th International Conference on Computers inEducation [CDROM]*. Hong Kong: Asia-Pacific Society for Computers in Education.

Sierpinska, A. and Kilpatrick, J. (1998) *Mathematics education as a research domain: A search for identity: An ICMI study*. Kluwer Academic publishers

Sinclair, P. and Varma, S. (2001). A relook at a mathematical bridge. In S. C. Agarkar, V. D. Lale (Eds.) *Proceedings of the CASTME-UNESCO-HBCSE International Conference*. Goa, India: HBCSE.

Subramaniam, K. and Naik, S. (2007) Extending the meaning of the fraction notation. In C. Natarajan and B. Choksi (Eds.) *Proceedings of epiSTEME-2 conference*. Mumbai: HBCSE.

Subramanian, J. and Verma, B. (2009) Introducing fractions using share and measure interpretations: a report from classroom trials. In K. Subramaniam and A. Mazumdar (Eds.) *Proceedings of epiSTEME-3 conference*. Mumbai: HBCSE.

Van den Heuvel-Panhuizen, Marja (2001) Realistic mathematics education as work in progress. In F. L. Lin (Ed.) *Common Sense in Mathematics Education, Proceedings of 2001 The Netherlands and Taiwan Conference on Mathematics Education (pp. 1-43)*. Taipei, Taiwan.