The Secondary Mathematics Curriculum: An Overview

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Some facts….

Diversity

- 42 recognized boards of education
- 3 national boards (CBSE, ICSE, NIOS)
- State boards function independently.

Enrolment

- 40 million students enrolled in secondary school in 2008
- Estimated enrolment - 60 million in the next decade

Challenges

- No availability of ICT in majority of schools
- Lack of professional development opportunities for teachers

Mathematics at Secondary Stage

Structure

- 10 + 2 system with 10 years of compulsory schooling
- Secondary stage marks the end of compulsory schooling
- Significant Landmark in the life of a student especially first generation learners - certification in secondary education by national or state boards
- Central focus – on consolidation of concepts acquired till the upper primary level and exploring wider connections

Mathematics at Secondary Stage

Content

- Emphasis on structure of mathematics and notion of proof
- **Number Systems** – going beyond calculations to understanding number systems in the abstract and exploring their properties
- **Algebra** – introduced in upper primary stage is developed at length Algebraic manipulations with linear, quadratic and rational expressions
- **Geometry** – reasoning about shapes and their properties using defined quantities and formulae
- **Trigonometry** – Understanding trigonometric ratios and identities
- **Elementary coordinate geometry**
- **Mensuration** - Visualizing solids, finding their surface areas and volumes, using algebraic skills to manipulate expressions and applying formulae.
- **Elementary Statistics**: Ability to represent, deal with and interpret data, Introduction to probability as a measure of uncertainty

Mathematics at Secondary Stage

Nature of Pedagogy

- Student is introduced to **formal mathematics**
- Introduction to **mathematical communication** – carefully defined terms, concepts, symbols
- Notion of argumentation and **proof** is an integral part of the curriculum
- Emphasis on **visualization, logical formulation** – linkages across topics such as importance of algebraic manipulations in geometry or trigonometry
- Reintegrate conceptual understanding and **consolidate conceptual knowledge** leading to problem solving ability .
- **Make wider connections** to other subject areas such as physical or social sciences

Mathematics at Secondary Stage

Assessment

- Largely of summative variety across boards
- CBSE made a major leap by moving towards formative assessment by implementing CCE (Continuous and Comprehensive Evaluation) which came into effect in 2009.
- CCE: An attempt at a paradigm shift from examination to effective pedagogy – no board exam at the end of class X
- Many state boards have shown inclination towards formative assessment

Mathematics at Senior Secondary Stage

Structure

- 10 + 2 system
- In class 11, the student has to make a choice of stream (Science – PCM,PCB, Commerce, Humanities)
- Significant Landmark: certification by national or state boards. School leaving examinations – high stakes examination – determines a students future
- NCF 2005 describes the senior secondary stage as the '*launching pad from which the student is guided towards career choices*'

Mathematics at Senior Secondary Stage

Content: Class XI	Content: Class XII
•Sets, relations and functions •Logic •Sequences and series •Linear inequalities •Combinatorics •Trigonometric functions •Straight lines •Conic sections •Complex numbers •Statistics	•Differential and Integral calculus (almost 50% of the curriculum) •Matrices and determinants •Vector algebra •Three dimensional geometry •Linear Programming •Probability

Mathematics at Senior Secondary Stage

Nature of Pedagogy

- Mathematics teaching - largely driven by preparation for the school leaving examinations.
- Curriculum makers have to make the difficult choice between depth and breadth. If breadth is chosen over depth, then the treatment of the topics should to be done at least to the extent that the student is able to see the relevance or utility of those topics in mathematics or in some other courses of study.
- Textbooks are examination oriented.

Mathematics: Senior Secondary Stage

Nature of Pedagogy

- In the school leaving examinations, students are tested only on the topics of class 12. Topics in class 11, rich in mathematical content, such as Sets, Relations and Functions, Sequences and Series do not get adequate attention.
- NCF recommends redistribution of topics.
- The approach to dealing with topics in textbooks have largely remained the same over the years. For example, the topic *Conic Sections* begins with a diagram showing the sections of a right circular cone. This is followed by definitions, derivations of the equations for different conics and their properties.

Mathematics: Senior Secondary Stage

Nature of Pedagogy: Sample questions from the textbook

- Manipulative and computational aspects of topics, rather than applications are emphasised.
- Some questions require the student to formulate or *model* a problem before applying a result, e.g
  - *A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without a lid, by cutting off a square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is a maximum?*
  - *A particle moves along the curve. Find the points on the curve at which the -coordinate is changing 8 times as fast as the -coordinate.*

Mathematics: Senior Secondary Stage

- Many questions are based on **direct application of formulae or rules** and these appear in examinations as well, e.g
  - *Find the angle between the straight lines* $y - x\sqrt{3} - 5 = 0$ *and* $y\sqrt{3} - x + 6 = 0$.
  - If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$

- Emphasis on questions requiring a substantial amount of **manipulative skills**
  - If $y = (\tan^{-1} x)^2$ show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$
  - Evaluate the integral $\int \sqrt{\tan x}\, dx$
  - Differentiate $\left(x + \frac{1}{x}\right)^x + x^{\left(x+\frac{1}{x}\right)}$

Mathematics: Senior Secondary Stage

Missed opportunities in the curriculum

- Lack of attention to connection between topics and across subjects is a major weakness of the curriculum.
- For example: the justification for the formula, *volume of a pyramid equals one-third area of base × height* can lead to an illuminating activity if the topic is revisited when the students have learnt about
- finding the sum of the squares of the first n natural numbers,
- proof by induction,
- Limits, obtaining the formula by a graded sequence of steps, starting with dividing the pyramid into slices of equal thickness; summing the volumes of the slices and estimating the limit computationally; and then determining the limit analytically

The Question of Technology

The Indian Context

- In many countries, there has been a shift of paradigm in mathematics teaching and learning due to powerful technology tools such as CAS, DGS, Spreadsheets and handheld calculators.
- In Indian classrooms
  - Traditionally the emphasis has been on computational algorithms and paper-pencil-drills.
  - General use of technology is not uncommon. But a very small number of schools use technology for mathematics instruction and if used, it is mainly for demonstration and does not engage the student actively.
  - The considered use of appropriate technology may help restore balance between the need for computational skills and the need for experiencing processes such as exploration and conjecture.

The Question of Technology

Challenges of Integrating Technology

- Technology must be **cost effective** and **easy to deploy** on a large scale
- **Teacher preparation**: developing sustainable professional development programmes for pre service and in service teachers which enhance skills in using technology and improve content knowledge using technology.
- A possible solution: setting up of technology enabled **mathematics laboratories** which provide the environment for learning mathematics through discovery, exploration, experimentation, modeling activities and investigatory projects.

Some Reflections

- The demands of mathematics education are changing, this needs to reflect in the mathematics curriculum in the way the topics are dealt with in the textbooks.
- The mathematics curriculum needs to provide inputs in terms of application to various fields of study such as the engineering sciences, biological sciences and even the social sciences.
- The use of real world applications and modelling in dealing with concepts in various topics will help create a context for applying mathematical theory.
- Mathematical modelling which challenge the student's conceptual understanding should be included in the curriculum more extensively. The models may deal with traffic flow, cryptography, optimization, genetics etc.

Some Reflections

- Aligning the senior secondary mathematics curriculum to the requirements of the undergraduate curriculum.
- The senior secondary mathematics curriculum needs to have adequate emphasis on the understanding of mathematics as well as problem solving skills. Presently the emphasis seems to be largely on the computational aspects.
- Topics which form the foundation of Pure Mathematics courses at the undergraduate level may be included in the curriculum at an elementary level such as Group Theory, Graph Theory, Game Theory, Markov chains and Numerical Methods.
- Move away from the ‘One size fits all’ curriculum. Some optional topics in the form of electives may be included.
- The senior secondary mathematics curriculum needs to undergo a major shift in terms of structure and presentation of content.