Teacher Knowledge of and Responses to Students' Thinking

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An Overview

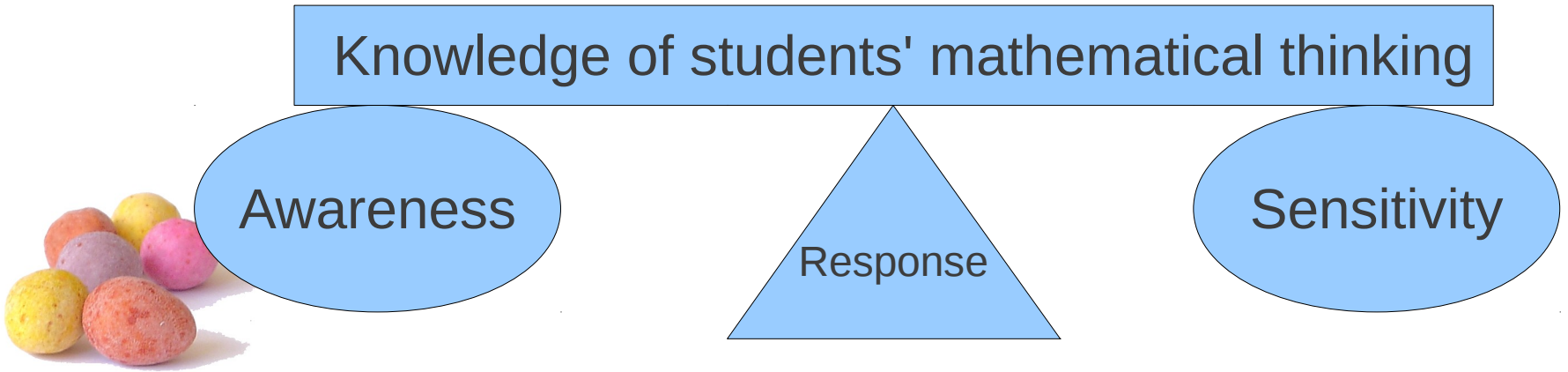
- Discourse in Mathematics Teacher Education
- Significance in Indian context
- Research Objectives
- Methodology
- Analysis and Findings
- Conclusions and Discussions

Framework for Teacher Learning

- Mathematical knowledge for teaching (Hill, Ball, & Bass, 2008), pedagogical content knowledge (Shulman, 1986), reflective practitioner (Schon, 1984)
- Teacher's **in-the-moment decision making** is a function of teachers' knowledge (Schoenfeld, 2011)
- Framework by Cochran-Smith & Lytle (2000)
  - Knowledge-for-practice
  - Knowledge-in-practice
  - **Knowledge-of-practice**

Knowledge about students' thinking

- Knowing ways in which students respond to different concepts (and sub-concepts)
- Understand and build on learners' prior knowledge and scaffold their learning by creating appropriate challenge
- Attentive, sensitive and responsive to students' ideas in relation to the concept being done
- Guides planning & in-the-moment decision making, creates opportunities for question asking, eliciting multiple strategies, drawing connections (Franke, Kazemi, & Battey, 2007)



In the Indian Context

- Disconnect between the knowledge of content and students
- Little focus on content specific students' knowledge required for teaching
  - Psychology and methods courses are separate
- National Curriculum Framework (2005) stesses on active learning and therefore teaching
- Need to set exemplar models of teacher learning

Investigation on...

- The study aims to explore
  - nature of teacher's knowledge about students' mathematical thinking & learning
  - relation between teacher knowledge & responses to students' mathematical thinking in & outside classroom
  - teaching practices which reflect manifestations of knowledge about students' thinking

    through
  - Design, implementation and evaluation of *classroom based tasks*

Methodology

- *Exploratory Case Study*
  - exploratory and interventionist
- Data Sources
  - Classroom Observations - video & audio, field notes
  - Task-Based Interviews - audio records
  - Studying Records - lesson plan, notebooks, etc.
- Participants and Settings
  - One teacher teaching two grade7 clasrooms with 34 students in each class, concept of Proportions
  - School encourages students' talking

Conduct of the study: Phase 1

Phase 1

Task based interview with the teacher (30 mins)

Classroom Observations (40 mins)

Analysis & Reflection

Task based interview with the students (20-30 mins)

Conduct of the study: Phase 2

Phase 2

Designing proportion problems for students using literature

Anticipation of students' responses with teacher

Problem-solving and its articulation by students

Reflection with the teacher

Data Analysis

Proportion Problem	Teacher Anticipation		Students' responses		Reflection with the teacher
	Strategy	Error	Strategy	Error	
The cost of 10 pens is Rs. 42. What will be the cost of 15 and 20 such pens?	Cross-multiplication	Cancellation Errors	Halving the cost of 10 and then adding it for 15. Doubling for 20. (S1,2,3,4,5)	None	The methods are good but they are common-sensical. I don't know how far will these methods help them. See he (S10) has done it using algebra. They [students] need to work systematically like this
	Algebra method (unknown as x)	Calculation errors	Cross-multiplication (S2)		
	Unitary method - find the cost of 1 and multiply it with 15, 20	Writing the ratios incorrectly like 10/41:x/15	Using the common factor 5 to find the cost of 15 and 20 pens (S6,11)		
	R: Don't you think students might use halving of doubling to solve this problem? T: I don't think they know that much. If I would have been at their place, I would not have used such long and complicated way. May be unitary method.		Algebra method (S10)		
			Unitary method (S7)		

Strategies used by students

It takes four people 3 days to wash all the windows of the K-Star mall. How long will it take for 8 people to do this job?

4 people 3 days
8 people ? x

24 = 8 × 3
4 × x
x = 6

8 people take 1 1/2 days because 4+4=8 ∴ then 3/2 = 1 1/2

1 1/2 day
4 people take = 3 days ∴
∴ 8 people will take = 3 ÷ 2 days

Since, There are more people the work will become easier.

∴ 8 people take = 1 1/2 day.

4 people 3 days
8 people

The no of people double therefore no. of days required will become 1/2

∴ 1/2 × 3 = 1.5 days or 1 1/2 days.

People	Days
4	3
8	?

1/2 day ≈ 8 people
4 + 4 = 8 people
3 days ÷ 2 = 1 1/2

4 people 3 days
8 people = 1 1/2 days

If we increase half People more then the days will get Half less.

Analysis and Findings 1

- Teacher's notion of formal mathematics aligns with her goals of teaching mathematics
  - *Math is about algorithms and problem solving, which are precise.. once the students have been taught algorithms, they are expected to use them while solving problems*
- Decisions like student selection to answer were based on personal traits of students
  - *He is hyperactive, does not pay attention in the class, so doesnot use the method properly*
- Strong content knowledge but not integrated with knowledge about students and their learning

Analysis and Findings 2

- Did not understand the significance of making students' thinking explicit while teaching in classroom
  - *Students' mistakes make me understand what errors they make but what else? Nothing else.*
- Teacher anticipation of students' thinking did not match with the students' responses to proportion problems
  - *They will not be able to solve inverse proportion problems as they have not been taught this*
  - *They would not use doubling and all, that is complicated. If I would have been at their [students'] place, I would have used unitary method*

An Example...

T- How to find the square root of 2025? To remove a square we put a square root on the other side. Now use the factorisation method [*teacher begins to do factorisation of 2025 on board*]
St- There is an easy method [*student repeats*]
T- I know
St- Can I show you the method?
T- No [teacher finds the factors of 2025 & makes pairs]
St- J (calling the teacher) You can directly do it
T- wait *[after 2 mins teacher leaves the class]*

[Student's strategy, interview after class]
St- we made 2025 from 25 & 81 (ref to the original ques). 5X5 & 9x9, so 5x9= 45, then why factorisation?

[Teacher's response in interview]
These are common-sense answers, they [students] are in school to learn algorithms. These answers will not help him in board exams.

Conflict creation…

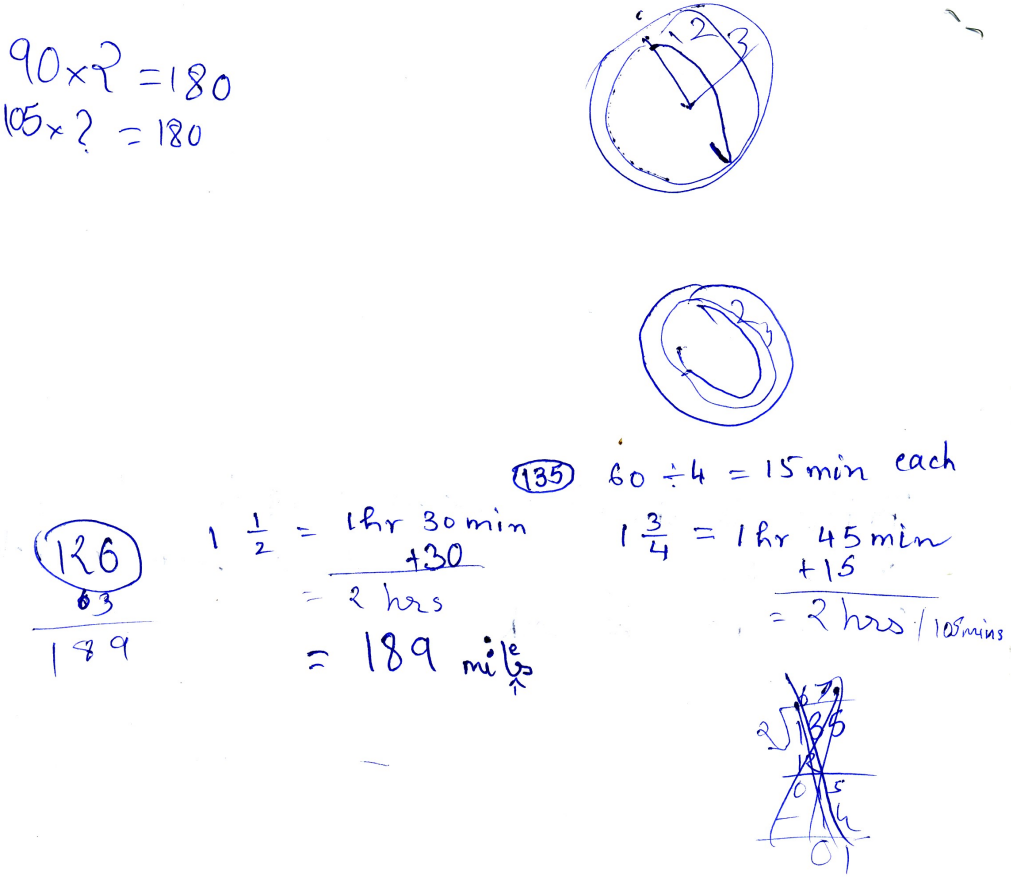
Which vehicle has faster average speed- a truck that travels 126 miles in 1½ hrs. or a car that travels 135 miles in 1¾ hrs.

- T: Solution is incomplete

  R: Let us try to solve the problem using S11 method

  Teacher and Researcher solve the problem using this method

  T: I never thought she could think like this



- Unaticipated responses and their analysis helped challenge teacher's thinking

Conclusions

- Students' thinking acts as a powerful context to elicit and support teacher knowledge in practice
- Teacher knowledge needs articulation, supports conflict creation – anticipation & reflection as a design to facilitate this process
- Knowledge of students' thinking is a significant for teachers and researchers
- Engagement with perspectives: methodological strength

- Extend the study to design, implement, & evalute classroom based tasks which can support teacher learning

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