Higher Secondary Mathematics Curriculum

Missed Opportunities

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20–22 Jan 2012

Part I

Inputs from NCF 2005

The Mathematics Focus Group of NCF 2005 made several observations and recommendations in its Position Paper.

It noted that for a math curriculum to be meaningful and effective, it must be coherent, focused on important mathematics, and well articulated across the grades. (Actually, this is how the NCTM put it.)

It put forth a vision statement which said (in part) that children must learn important mathematics, and pose and solve meaningful problems.

In its observations on “Problems in Teaching and Learning of Mathematics” NCF highlighted the ill effects of a curriculum that emphasizes procedure and knowledge of formulas over understanding.

It recommended that we shift the focus of mathematics education from achieving narrow goals to higher goals.

What are these ‘higher goals’?

Where are we placed with regard to them?

Higher goals

Attention to be given to . . .

- Formal problem solving
- Heuristics
- Estimation of quantities
- Approximating solutions
- Optimization
- Use of technology
- Visualization
- Representation
- Making connections
- Mathematical communication
- Mathematics that people use
- Humanizing the curriculum

Depth versus breadth

NCF 2005 also addressed the important question of depth and breadth of the curriculum. Here are some consequences of what it said.

- If a theme is introduced, then it should be developed to the extent that its links with other themes and topics are seen and understood.
- Exposure to the theme should empower a student and add to his or her understanding in some significant way.
- If this is not possible, the theme is best not introduced.

Part II

CBSE Math Syllabus, 11–12 (2012)

[Preamble.] To acquire knowledge and critical understanding, . . . apply skills acquired to solve problems, . . . acquaint students with math used in daily life, . . . develop an interest to study math as a discipline, . . . develop a feeling for national integration, protection of environment, ...

Class 11 (2012)

- ***Sets and Functions.*** Sets, relations, functions, trig functions
- ***Algebra.*** Induction, complex numbers, quadratic equations, linear inequalities, counting, binomial theorem, sequences, series
- ***Coordinate Geometry.*** Straight lines, conic sections, 3-D geometry
- ***Calculus.*** Limits, derivatives
- ***Mathematical Reasoning.*** If and only if, implies, and/or, there exists, contradiction, converse, contrapositive
- ***Statistics and Probability.*** Measures of dispersion, probability

Class 12 (2012)

- ***Relations and Functions.*** Equivalence relations, composition of functions, inverse function, binary operation, inverse trig function
- ***Algebra.*** Matrices, determinants
- ***Calculus.*** Continuity, differentiability, mean value theorems, integrals, applications of derivatives and integrals, differential equations
- ***Vectors and 3-D Geometry.*** Vectors, line and plane in 3-D
- ***Linear Programming.*** Graphical solution of 2-D problems
- ***Probability.*** Multiplication rule, conditional probability, random variable, mean, variance, binomial distribution

Changes from earlier years

Topics which were studied in 1989 but are no longer in the syllabus

- ***Mathematical Logic.*** Truth tables, conditional statements, switching circuits, . . . (This topic was removed, then re-introduced in 2003 under the chapter *Boolean Algebra*, but this too was dropped in 2005.)
- ***Correlation and Regression.***
- ***Computing.*** Computer arithmetic, programming languages
- ***Numerical Methods.*** Numerical solution of polynomial equations and systems of linear equations

Changes from earlier years (continued)

Linear programming and *commercial arithmetic* were introduced in 2000.

(The latter was removed at some point.)

In 2005–07 the NCERT math textbooks were revised (post NCF 2005).

The textbooks now have two chapters in the Appendix:

1. *Proofs in Mathematics*
2. *Mathematical Modeling*

Part III

Wasted Effort?

Could we describe any of these topics as 'wasted effort'?

Do they contribute to the 'inch deep, mile wide' syndrome?

Yes . . . and here are some candidate topics

- ***Continuity, differentiability*** (in both CBSE and ISC syllabuses)
- ***Mean value theorems*** (in both CBSE and ISC syllabuses)
- ***Matrix algebra*** (in both CBSE and ISC syllabuses)
- ***Descriptive statistics*** (in most syllabuses)
- ***Correlation*** (in ISC syllabus, not in CBSE syllabus)
- ***Boolean algebra*** (in ISC syllabus, not in CBSE syllabus)
- ***Group theory*** (once in ISC syllabus; still in some state syllabuses)

Sample questions

1. Verify Rolle's theorem for the function $f(x) = \sin x$ defined over the interval $[0, \pi]$.
2. The following table gives two kinds of assessment of the work of ten students. Find Spearman's coefficient of rank correlation and interpret the result. [Table follows.]
3. Prove the identity $1 + a = 1$ in a Boolean algebra.
4. Show that $\{1, 2, 3, 4\}$ forms a group under multiplication modulo 5.

An important clarification . . .

We are not commenting on the mathematical importance of these topics, or their intrinsic beauty.

We are commenting on the manner in which they are studied, and the (lack of) depth, which make rather a mockery of their inclusion.

Recall what NCF 2005 said about (the effects of) "*. . . a curriculum that emphasizes procedure and knowledge of formulas over understanding.*"

Also, its comment: If a topic is introduced, it should be developed to the extent that its links with other topics are understood.

Part IV

Missed Opportunity?

Are there opportunities in the syllabus, where without any further additions being made to the syllabus, much more can be done?

Where connections can be shown in a natural and organic way?

Where technology enabled exploration is possible?

There is one main goal of mathematics education in schools — *the mathematisation of the child's thought processes*. . . .

It is "more useful to know how to mathematise than to know a lot of mathematics" [David Wheeler].

— Adapted from NCF 2005

Volume of a right circular cone

- $V = \frac{1}{3}\pi R^2 h$
- Proof using division into n slices of thickness h/n, treating each slice as a disk
- Use of formula for $1^2 + 2^2 + \cdots + n^2$ to estimate volume of cone
- Occurrence of a limit; using technology to estimate the limit; guessing the limit, then proving it using the summation formula
- Proof using integration
- Bringing it all together: class 10, etc

Volume of a right pyramid with square base

- $V = \frac{1}{3}(\text{area of base} \times \text{height})$
- Notion of a shear, which leaves volume unchanged; Cavalieri principle
- Division into n slices of thickness h/n; treating each slice as a cuboid with a square base; use of formula for $1^2 + 2^2 + \cdots + n^2$; occurrence of a limit; using technology to estimate the limit; etc
- Proof using geometric transformations (shears, etc)
- Math lab activity: 6 pyramids and a cube
- Bringing it all together

Volume of a sphere

- $V = \frac{4}{3}\pi R^3$
- Division into n slices of thickness h/n; treating each slice as a disk; use of formula for $1^2 + 2^2 + \cdots + n^2$; etc
- Bringing it all together: class 10, etc

Triangular numbers

- Exploration of the sum $T_n = 1 + 2 + \cdots + n$
- Guessing a formula for T_n and proving it is a wide variety of ways: induction, summation of an AP, etc
- $T_{n-1} + T_n = n^2$
- Proof using straightforward algebra
- Visual proof
- Proof using counting
- Similarly for $8T_n + 1 = (2n+1)^2$

Integer factorizations using algebra

- $a^2 - b^2 = (a-b)(a+b)$
- Fermat's method of factoring
- $x^4 + 4 = (x^2 - 2x + 2) \cdot (x^2 + 2x + 2)$
- "Factorize the numbers $5^4 + 4 = 629$, $15^4 + 4 = 50629$ and $25^4 + 4 = 390629$."
- $x^8 + x^4 + 1$ has $x^2 + x + 1$ as a factor
- "Factorize the numbers $3^8 + 3^4 + 1 = 6643$ and $6^8 + 6^4 + 1 = 1680913$."

More possibilities

- Bijective proofs of simple identities, e.g., $(n+1)^2 = n^2 + 2n + 1$, $\sum_{k \geq 1} k \cdot \binom{n}{k} = n \cdot 2^{n-1}$, or even $1 + 2 + 3 + \cdots + n = \binom{n+1}{2}$
- Déscartes-Euler formula, $V - E + F = 2$ (as an exercise in induction)
- Number theoretic consequences of the formula $|uv| = |u||v|$ where u, v are complex numbers; example: integers which are sums of two squares ($5 \times 13 = 65$)
- Geometric consequences of factorization of $z^n - 1$ (Example: For a regular n-gon inscribed in a unit circle, product of the distances from one vertex to the remaining vertices equals n)

. . . And still more

- Trigonometric proofs of some geometric theorems: Euler's theorem (collinearity of O, G, H); Hero's formula for area of a triangle, and Brahmagupta's formula for area of a cyclic quadrilateral; Napoleon's theorem; Ptolemy's theorem
- Newton-Raphson algorithm for solving equations
- Bakhshali square root formula
- Curve sketching (an enormously rich topic, bringing in strands from algebra, limits, derivatives, . . .)