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BELIEFS AND ATTITUDES TOWARD MATHEMATICS AT UNIVERSITY LEVEL, DEVELOPMENT OF MATHEMATICAL KNOWLEDGE

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Mathematics is increasingly useful in modern society. Citizens need to learn mathematics to be competent human beings. However, many Mexican students dislike mathematics, in particular those who are studying social careers. In this article we report partial results of a research project focused on the development of mathematical knowledge. We use Problem Solving (PS) and Models and Modeling (MM) perspective to encourage students to learn mathematics. The procedures of students to solve problems show the mathematical knowledge, beliefs and attitudes exhibited toward the discipline. The reflection emerges because of the introduction of PS and MM Perspectives at the university courses since 2010.

Beliefs, Attitudes, Mathematical understanding, ways of behavior

INTRODUCTION

We have the task of designing university courses on mathematics for students who are starting the studies (first semesters) at the University, in particular those who are pursuing social carriers. About 80% of these students said they dislike mathematics, they feel that they are not so good in it and they elected a carrier where fortunately they won't study more than one or two mathematics course.

The main content of one of these university mathematics courses (named Mathematics I) that students have to learn is algebra and functions. There are four units in the program (Algebra, Progressions, Functions and Statistics), but the semester is not enough to discuss all the units, so the students only learn three of them (A, P, F). Function is the concept that unifies the knowledge in this Mathematics 1 course.

The mathematical knowledge background of the students is from high school. The Mexican high school mathematics programs include algebra language, linear and quadratic equations, system of linear equations (SEL) and functions. Some methods that students have to learn are elimination, substitution, Cramer's rule and graphing to solve linear of system equations up to three unknowns, they have to learn how to elaborate tables, graphs, and symbolic expressions. However, we have found that students at the university level have difficulties to use tables, graphs and symbolic expression to solve problems. The students don't relate these representations with the concepts of function and equations.

In this article we report about the ways of behavior, models or ways of thinking, which include mathematical knowledge, beliefs and attitudes that students exhibit when they solve problems related with concepts as equation, function, variable, unknown and solution, at the university level of education. What kind of mathematical abilities and knowledge do students exhibit in the procedures to solve a problem? What kind of beliefs and attitudes are related with? We try to answer these questions using the findings draw of a research project focused on the development of mathematical knowledge related with the concepts as equation, function, variable, unknown and solution.

LITERATURE REVIEW

The learning of function and linear equations is a topic that has been studied since several years ago, as part of the research of algebra in mathematics education. The meaning of algebra in the schools was extended since memorizing rules, manipulating symbols and solving equations toward the learning of concepts as functions, variables, unknowns, relations, patterns and generalizations. Kieran (2006) described three tendencies of research in algebra where the meaning of algebra, the learning, and the approaches to research the teaching and learning have been evolving. For example, the use of technology has been used as a mean to understand concepts, relations and process. The language and discourse used by students when they solve problems is now considered important.

The activities, situations and problems used to learn and teach mathematics are essentials to construct and modify meanings. The learning is closed related with the creation of models that permit interpret, describe and explain situations or systems (Lesh, 2010). The conceptual systems (or models) created can be internal (to individual) or external (to communicate). There is an interaction and interdependency of these two systems. They are not separated. The development of internal models is influenced by external models, and the communication of external models influence internal models.

> It is sometimes tempting to think of a model reside in a particular set of mathematical symbols (e.g. in the symbols of a particular functional relation) that are external to that individual. However, the meaning of those symbols, we argue resides partly in the head of the individual and partly in the symbols". (p. 233, Doer and Trip, 1999)

The students participate in the school learning activities based on their beliefs and understanding. We can observe in the classroom the students´ beliefs about their role and others' roles, about the general nature of mathematical activity, mathematics beliefs and value and mathematical conceptions (Yackel & Rasmussen, 2002). All beliefs are related among them. The beliefs, attitudes and understanding are the product of students' mathematical experiences and they can be modified by the interactions with environments.

The student development typically occurs through participation in learning communities (Lesh & Yoon, 2004). The communities and practices in which individuals participate or involve by thinking about mathematical topics and discussing ideas are important. The conceptual development of individuals is influenced by shared constructs, conceptual tools, and social norms of the group.

Beliefs, attitudes and understanding are related in a complex way; they are inside of conceptual systems (models). The students have the opportunity to change them when they have to test, modify and refine the models in the community. At the same time, the students interiorize ways of thinking that are valued to the community when they solve problems. These ways of thinking become part of the practice of the students.

METHODOLOGY

The students who took part in this study were 18 years old. They were a group of 20 students enrolled in the first year of university level. The group of students was starting social science careers such commercial systems. So, the mathematical knowledge background was from high school.

The problems described in this article form part of a series of seven problems and situations designed in a research project focused on the development of mathematical knowledge related with concepts as equation, function, variable, unknown and solution. The students solved the problems in teams of three and four students. The role of the teacher was to create an environment where students had to discus, argument, share, and explain the conjectures and procedures in the classroom. Advances were discussed at various times by the teams. The students had to write final individual reports as homework. Some of the goals of writing final reports were that students had the opportunity to organize their reasoning ways, to identify mathematical knowledge involved and to have a holy vision of the solution of the problem.

All the problems required the concepts of function, equations, variables and solution to analyze the situations involved. The students needed to construct tables, graphs and equations to describe, explain and predict. Two of the problems and situations that are reported in this paper are the following.

Problem 1. Analyze how the company charges for shipping DHL packages nationally and how the price varies on the weight of the package.

We can reduce the analysis to answer the following:

1. How would you describe the process of charging the company for a given zone?
2. What price difference between sending a 1 kg package and 2 kg package?
3. How much will I be charged for sending a packet of 2kg and 300 g?
4. Is it the same price difference between sending 3kg and 4kg than between 5 kg and 6 kg?
5. What is the behavior of these differences?

The students received information about tariff zones in the country and package shipment prices, depending on the weight of it (Data was taken from: http://www.dhl.com.mx/es/express/centros_de_recursos/tarifas_servicios_nacionales.html#time_definite). We asked students to analyze information to explain to a worker of the company how to calculate the cost of a shipment of packages for a given area, even if the weight were not part of the table of data. They had to analyze the information and they had to observe variations.

The description of the variation in the cost of sending packets for a given area could be done through a process of graphing, which would lead to a stepped function. The students also could symbolize the relationships in algebra language.

> Problem 2. A photocopying machine is $ 20 000.00 and produces 1000 copies per hour, with a production cost per copy of $ 0.1. Another photocopying machine has a cost of $ 40 000.00 and produces 1500 copies per hour; with a production cost of each copy equals $ 0.05. How many hours should use machine most expensive for it to be affordable? When is it more convenient to buy one of photocopier than the other one?

The system of linear equations (SEL) that represents this problem is as follows:

$$\begin{aligned} 100x - y &= -20000 \\ 75x - y &= -40000 \end{aligned} \quad (1)$$

The total production cost, considering the cost of the machine is y and the number of copies is *x*. 100x and 75x come from the production costs of photocopiers 1 and 2 (1000*0.1 x *and* 1500* 0.05 x), regardless of the initial cost of photocopiers (20000 and 40000). The solution of the SEL is: $\boldsymbol{x} = 800, \quad \boldsymbol{y} = 100000$

The second problem was closer to the problems that students used to solve in the classroom at high school. The description of total cost could be done through a process of graphing to answer the second question. The students can also write inequalities.

In this paper we use the procedures used by the students to solve problems 1 and 2 to illustrate students' ways of behavior, models or ways of thinking. The main focus we are interested to show is ways of behavior and models or ways of thinking in the process of solving problems that students exhibited in the classroom and also in the reports they did as homework. These ways of behavior are characterized by beliefs, attitudes and understanding of mathematical concepts. The arguments used by students were very important. They gave information about the development of knowledge and the process to construct meaning about mathematical knowledge related with concepts as function, equation, variable and solution. The data that we analyzed were taken from final reports done by students and the teacher's bitacora.

RESULTS

We identified ways of behavior or ways of thinking at the beginning of the course that were changing at the same time as students solved the problems and discussed mathematical knowledge in the classroom. Some of these behaviors are mentioned by literature. The most important idea here is to show how these ways of thinking include mathematical knowledge, beliefs and attitudes that students exhibit when they solve problems related with concepts as equations, function, variable, unknown and solution, at the university level of education course of Mathematics 1.

First way of behavior

The first attitude students exhibited, when they began to solve the first problem (Problem 1) in the course, was to wait that teacher gave specific instructions to solve it. The students asked the teacher: What do I have to do?

For example, in Problem 1 students were confused and they didn't know what to do. What did the word *analyze* mean in mathematics classroom? The attitude was to wait teacher instructions as: write equation, do a graph, do a table, solve it using similar procedure. The students observed so much information and they didn't use any heuristic to analyze it. Many students believed that mathematics problems only ever have one solution, so solving problems meant to find that solution characterized by one numeric value.

In Problem 2, the situation was similar the students asked: What do I have to do? The students could not retrieve heuristics to analyze de situation. The role of the teacher in both problems was to problematize the activity; he used questions as: do you understand the problem or situation? How can we answer the questions? What do we have to do to find the solution? The teacher suggested the use of whatever procedure that could help to find the answer or solutions: arithmetic, graphs, or symbolization.

Second way of behavior

The following behavior that students exhibited when they were solving Problem 1 was related with validation. How could I know if my procedure or answer is right? The students didn't come back to the problem text and verify if they were answering the questions. They asked the teacher to validate the procedures they were doing. Also, several students believed that mathematical procedures should be algebraic because "university students have to use algebra expressions to solve a problem"

In Problem 2, for example, the students were looking for data and quantities in order to relate and symbolize them as equations. Algebraic procedures emerged, but not in an appropriate way, because students tried to guess the kind of relations among quantities. They looked for something to represent with variables *x* and *y*. Students identified questions established in the statement of the problem, but during the process they forgot to answer them. In this stage students analyzed part of the given information in the text of the problem as questions, data and relationships among them. All the students found the cost of copies that photocopiers could produce in one hour (Figure 1). They calculated the total cost without considering the prize of the photocopier machine.

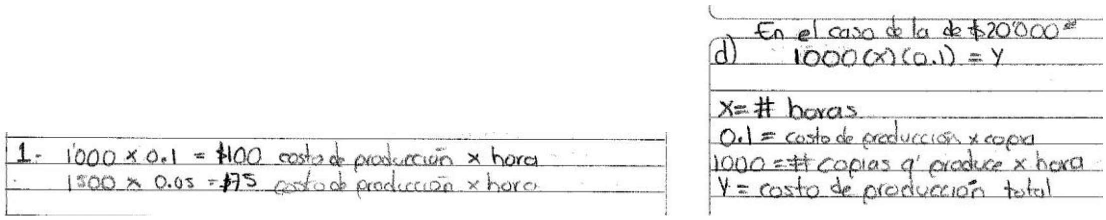

Figure 1. Procedure to solve Problem 2

The students could analyze how this cost depended of the hours, but they couldn't use the time to compare the cost of production of photocopiers. They didn't consider the time (one hour, two hours of using the machine) as criteria to compare the cost of production, the students instead divided 20000 by 100, for example.

In Problem 1, because students didn't receive any type of specific instructions, they read all the information and tried to understand the process to find the cost of sending any package. They focused the attention to understand the information included in the different tables that the problem included.

Third way of behavior

The third behavior we could observe was that students used different representations, they looked for patterns and they wrote symbolic expressions. The role of the teacher was here important to help students to encourage them to use different representations to understand the situation and to organize the information.

In problem 1, the students began to work with arithmetical and graphical procedures to analyze the situation. Teams of students who were using arithmetical procedures didn't use graphs and vice verse. Some students for example took data from the tables and made graphs. In problem 2, all the teams of students worked with arithmetical procedures. The teacher validated this procedure and encouraged them to look for patterns (Figure 2). Graphical procedures were a suggestion given by the teacher, but students didn't consider it very important (Figure 3).

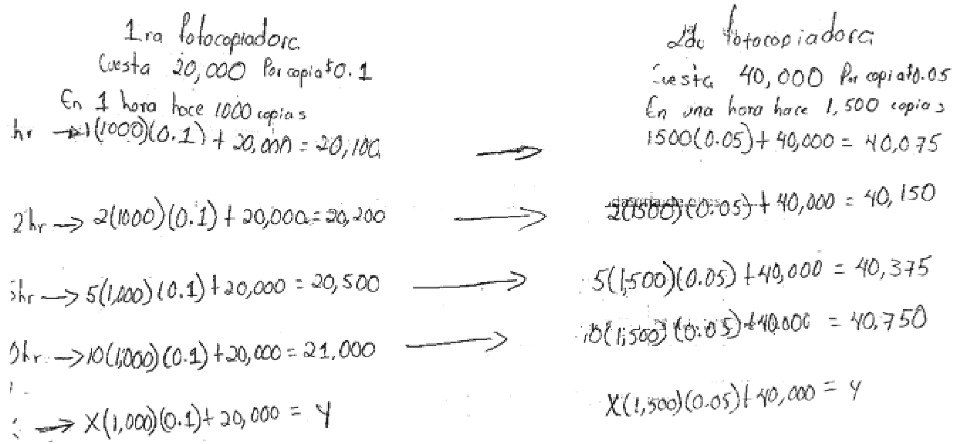

Figure 2. Arithmetical procedures to solve Problem 2.

The role of the teacher was to discuss in the classroom the solution of the problem and the different procedures to represent it. Some questions that emerged in the classroom were: How to connect the representations to understand a situation? How representations could give us different information? Which graph is the best to represent the situation? Could these representations help to answer the questions of the problem? Which were the questions in the problem? ¿How did we understand the problem? ¿In which moment did we get to understand it? The students discussed these questions and arrived to conclusions. The homework was to write a report (Figure 3).

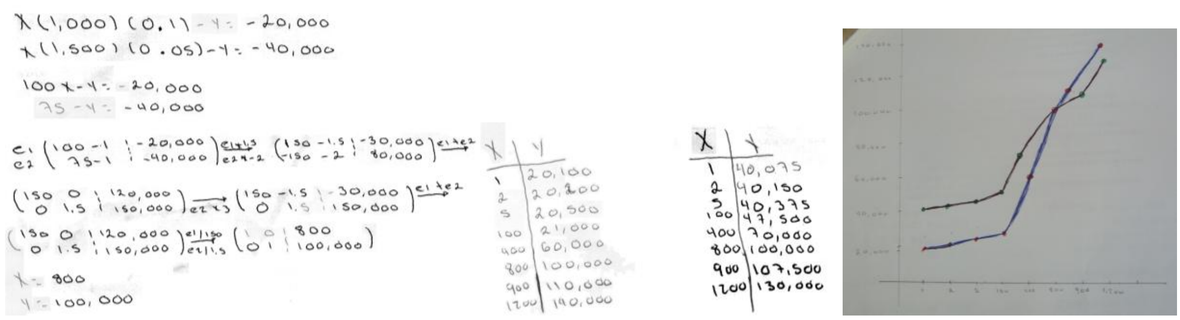

Figure 3. Procedure to solve Problem 2. The students didn't argument the relation among procedures.

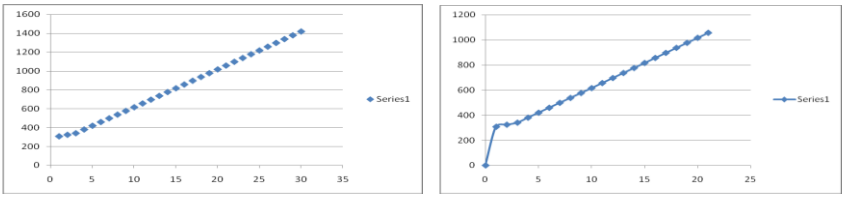

Figure 4. Procedure to solve Problem 1. The students didn't construct the stepped function by themselves. It was the product of the discussion and they did it using paper and pencil.

We have some observations here. At the beginning of the course, students didn't validate and didn't recognize graphs and tables as useful mathematical procedures to understand problems, neither as representation of function concept. Arithmetical procedures didn't seem mathematical knowledge. Graphical procedures didn't seem important. The students didn't understand the connection among representations. Algebraic procedures were the better form to solve a problem, although students couldn't use it to have a complete interpretation and description of the situation.

The students tried to reproduce the procedures discussed in the classroom as homework. The reports were like a summary of the activity in the classroom (figures 2 and 3), however they made some errors; for example, the graph of Figure 2. Another team introduced graphical representations as an annex to the final report. The students didn't argument the representations and the relations among representations. They only wrote operations and added tables and graphs. They didn't justify the use of them. They underlined the solution without explain or argument it. Even when the professor asked about the connection among representations, they couldn't explain it.

Along the course the students were interacting with problems where concepts as function, variable, equation and solution were involved. The students were analyzing information using arithmetic, looking for patterns, writing equations, doing tables and graphs. These representations became to be interiorized by students not only as procedures to solve a problem, also as representations of the concept of function. The analysis in terms of "what can this model describe, explain and predict?" to compare procedures in the classroom was useful to validate representations and the relation among them. During the course the meaning of concepts as function, variable and solution were changing.

At the end of the course, in the final reports, the students became to modify the content. They tried to argument procedures. They tried to connect the representations and explain the situation in finals reports. Students validated the arithmetical and graphical procedures to understand a situation. The knowledge about how to graph and symbolize was better. The students began to understand that a function can have different representations. They also looked for answering the questions in the problems.

DISCUSSION OF RESULTS AND CONCLUSIONS

The research questions posed in this paper were what kind of mathematical abilities and knowledge do students exhibit in the procedures to solve a problem? What kind of beliefs and attitudes are related with?

The students started the university courses with basic mathematical knowledge and procedures that could be used to solve problems. However, concepts as function, variation, equation and solution were teaching in high school as separated topics, without connection among them. The belief about that arithmetic and graphs were not relevant as procedures in mathematics to solve a problem influenced the development of the knowledge about function. Expressions as “the beautiful of the mathematics is that we only have to calculate, we don’t have to write” told by good students influenced the process of argument a solution and procedures. All the students had difficulties with the process of justification and argumentation at the beginning of the course.

The use of several representations to solve and analyze problems helped students to have better understanding not only about situations, also about the concept of function. It also helped to change the beliefs and attitudes to some procedures as arithmetic and graphs. The community permitted that students were incorporating practices to solve problems. Discussions in the classroom, revision of the homework by students and the process of communication were useful. But the most important aspect was the definition of criteria considered to solve problems. The students were modifying and refining the Models and ways of thinking during the process of problem solving.

Some attitudes and beliefs that appeared during the course need time to be modified, for example, the students continue thinking that they need to understand very well mathematical concepts before applying to a situation. They couldn’t see that they can deep understanding while they are solving problems. The anxiety continues in the students who sometimes prefer doing exercises than solving problems, but the motivation to learn and use mathematics in the classroom is better when students have to solve and analyze problems and real situations. Creating models and ways of thinking during the process of problem solving give the opportunity to students modify beliefs, attitude and understandings.

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